# SIMULATION OF FLUID-STRUCTURE INTERACTION USING SMOOTHED PARTICLE HYDRODYNAMICS AND NONLINEAR SHALLOW WATER EQUATIONS

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## ABSTRACT

The present paper presents a numerical model based on the Lagrangian approach of Smoothed Particle Hydrodynamics (SPH) that is used to solve the Nonlinear Shallow Water Equations (NSWE). The model has been developed within the framework of a research project aimed at studying tsunamis generated by landslides (www.tsunamis.it). The mathematics of the numerical model along with the numerical algorithms are here presented. Then, applications of the model to a dam break wave hitting a tall structure and to a standard benchmark case of tsunamis generated by landslides are shown. For both cases experimental results are available for comparison with the computations. The model seems to be able to reproduce with good accuracy and at the cost of very low computational time the flow in the investigated conditions.

*Keywords*: Smoothing Particles Hydrodynamics, SPH, tsunamis, landslide generated waves

## 1 INTRODUCTION

The Smoothed Particle Hydrodynamics (SPH) approach is here used to model the Nonlinear Shallow Water Equations (NSWE) with the aim of setting-up a model to properly reproduce tsunamis waves and their interaction with the coast. The present study has been conceived within the framework of a wider research program on tsunamis waves, funded by the Italian Ministry of University and Education MIUR, by the Italian Civil Protection Agency and by the National Dam Office. The program, which is carried out at the LIAM laboratory of L'Aquila University, aims at characterizing tsunamis by means of studies on physical models and numerical simulations. A major task of the research program is the development of a Tsunamis Early Warning System (TEWS) where the present NSWE-SPH numerical model will be used to simulate tsunamis wave run-up at the shoreline.

Numerical models based on NSWE have been widely used to simulate several problems in hydraulics and coastal engineering, such as flooding waves, propagation of impulse waves downstream a dam, and also tsunamis waves. The two-dimensional shallow water equations are obtained by depth-integrating the continuum mass and momentum balances given by the three-dimensional incompressible Navier-Stokes equations, under the hypothesis that the waves are much longer than the local water depth. The NSWE are commonly solved using numerical schemes based on finite differences, finite volumes or finite elements techniques. However, ad hoc method are always used at the shoreward boundary of the computational domain to simulate the wave run up, i.e. the instantaneous movements of the shoreline. Different techniques have been introduced to simulate waves run up in depth-integrated models (NSWE and Boussinesq-type models), some of which are reviewed in *Bellotti and*

*Brocchini* (2001) and *Brocchini et al.* (2001).

Here we use a different numerical approach, Smoothed Particle Hydrodynamics, which has the point of strength of being able of easily dealing with complex and moving boundaries of the computational domain. Here we take advantage of this ability of SPH and we use NSWE to reproduce flow fields characterized by fluid interacting with structures. The water columns are simulated using a lagrangian scheme and are free to evolve their positions without any limitation.

Basic assumptions of SPH models are that the continuum is made up of a finite number of particles interacting to one another, evolving their position in a lagrangian meshless scheme. Continuity and momentum equations are rewritten considering a kernel function $W$ which is used to weight the particles interaction and reciprocal contributions. During the last decade, SPH has been applied to simulate free surface flows (*Monaghan*, 1994; *Gallati and Braschi*, 2000), multi-phase flows (*Colagrossi and Landrini*, 2003), tsunamis waves (*Monaghan and Kos*, 2000; *Panizzo*, 2004; *Panizzo and Dalrymple*, 2004). Advances in lagrangian numerical models of NSWE have been gained in recent years using SPH by *Gallati and Sturla* (2001), *Frank and Reich* (2003), *Rodriguez-Paz and Bonet* (2005) and *Soulaimani and Ata* (2005). In this paper we introduce a numerical scheme able to deal with particles with different volumes, letting us to set the particles layout on a regular grid even considering complex bottom geometries, thus improving the model resolution at the shoreline.

In the following we present the 1D and 2D versions of the SPH-NSWE numerical model. Then, SPH-NSWE is applied to simulate benchmark cases from laboratory studies, underlining the model's points of strength and weakness. In particular, we apply the model in a case test where surely 3D like models, i.e classic SPH and the VOF, should be more properly indicated to simulate a complex three dimensional flows. Indeed we show that SPH-NSWE results reasonably agree with experimental observations, at a very low computational time. Finally, conclusions and brief remarks on ongoing researches are discussed.

## 2 MATHEMATICAL FORMULATION OF NSWE IN SPH APPROACH

The Nonlinear Shallow Water Equations can be written in different forms. Making reference to notation and symbols presented in Figure 1 and neglecting in this expression the bed friction that will be later considered, here we use the following form

$$\begin{cases} \dfrac{\partial \eta}{\partial t} = -\nabla \cdot \mathbf{v}(\mathrm{h} + \eta) \\ \dfrac{\partial \mathrm{v}}{\partial t} = -\mathbf{v} \cdot \nabla \mathbf{v} - g \nabla \eta \end{cases} \tag{1}$$

where $\mathbf{v} = (u, v)$, is the depth integrated water velocity, $h(x,y)$ is the bottom elevation, $\eta$ is the surface displacement, and $g$ represent the gravitational acceleration. If we consider total derivatives of (1), and we refer to $d_{tot}$ (i.e. summation of the bottom elevation $h(x,y)$ and the surface displacement $\eta$) and $\mathbf{v}$ as model variables to deal with, applying a Reynolds transport theorem we can rewrite the system of NSWE in Lagrangian form as

$$\begin{cases} \dfrac{Dd_{tot}}{Dt} = -d_{tot}(\nabla \cdot \mathbf{v}) \\ \dfrac{D\mathbf{v}}{Dt} = -g \nabla d_{tot} + g \nabla h \end{cases} \tag{2}$$

With the aim of taking into account bottom friction, we rewrite the second equation of system (2) as

$$\frac{D\mathbf{v}}{Dt} = -g\nabla d_{tot} + g\nabla h - g\mathbf{S}_f \tag{3}$$

where $\boldsymbol{S}_f=(S_{fx}, S_{fy})$ is the Manning formulation of the bottom friction, expressed as

$$\mathbf{S}_f = {n^2|\mathbf{v}|\mathbf{v}}/{h^{4/3}} \tag{4}$$

where $n$ is the Manning coefficient assumed equal to 0.01 (s/m$^{1/3}$).
If we rewrite the system of equations in (2), we now have

$$\begin{cases} \dfrac{Dd_{tot}}{Dt} = -d_{tot}\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) \\ \dfrac{Du}{Dt} = -g\dfrac{\partial d_{tot}}{\partial x} + g\dfrac{\partial h}{\partial x} - gS_{fx} \\ \dfrac{Dv}{Dt} = -g\dfrac{\partial d_{tot}}{\partial y} + g\dfrac{\partial h}{\partial y} - gS_{fy} \end{cases} \tag{5}$$

In this scheme the model variables are the horizontal velocity components $u$ and $v$, and the total water depth $d_{tot}$. Water column position $x$ and $y$ are updated according to velocity values. In the SPH formulation of the NSWE, each particle represents a water column with a given volume and mass, kept constant during the simulation. If we refer to classic SPH approaches simulating water with a certain number of compressible particles, we can say that the variability of particles density in that case is similar to the variability of water column total depth $d_{tot}$ in this case. Indeed, we can face the SPH-NSWE model assuming a close analogy with classic SPH models.
Figure 1 reports how the water column and its physical variables are represented in the lagrangian particle approach, being each water column represented by a particle interacting with its neighbours in a meshless scheme. Particles (water columns) interaction is governed by a variable smoothing length that is necessary to ensure the correct number of neighbours also in the case that the water column height evolve far from the initial condition. The proper number of neighbouring particles should be about 5 and 21 for 1D and 2D case respectively.

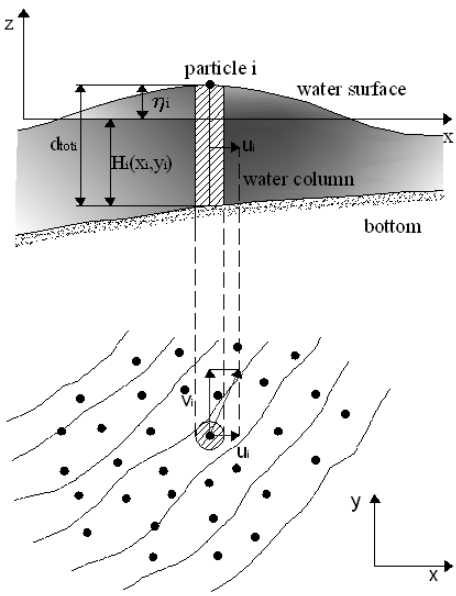


**Figure 1**. Lagrangian sketch of particles representing water columns in the NSWE-SPH approach.

The updated value of the interaction radius is expressed as

$$l_{i,n} = l_{i,o}\left(\frac{d_{tot_{i,o}}}{d_{tot_{i,n}}}\right)^{1/dm} \tag{6}$$

where $d_{toti,0}$ and $d_{toti,n}$ are the particle's total depths, $l_{i,0}$ and $l_{i,n}$ are the particle's smoothing lengths at the beginning of the simulation and at the generic time $n$, $dm$ is the number of dimensions.
In the classic SPH approach each physical variable $A$ of the flow field can be expressed as a finite summation of the same variable over neighbouring particles, such as

$$A_i = \sum_{j=1,N} A_j W_{ij}(\mathbf{x}_{ij}, l_i)\frac{m_j}{\rho_j} \tag{7}$$

$m_j$ and $\rho_j$ being the particle's mass and density, and $W$ is the kernel smoothing function which weighs the particles contribution to the particle (or the generic point) at hand $i$. In the SPH-NSWE approach, equation's variables (7) can be expressed as

$$A_i = \sum_{j=1,N} A_j W_{ij}(\mathbf{x}_{ij}, l_i)\frac{V_j}{d_{tot_j}} \tag{8}$$

Consequently we can write the total water depth at a certain point as

$$d_{tot_i} = \sum_{j=1,N} V_j W_{ij}(\mathbf{x}_{ij}, l_i) \tag{9}$$

which is an implicit equation as $l_i$, like reported in (6), is expressed as a function of $d_{toti}$. Variation in time of $d_{toti}$ can be evaluated with the following equation

$$\frac{d\left(d_{tot_i}\right)}{dt} = \frac{d_{tot_i} dm}{\omega_i}\sum_{j=1,N} V_j \nabla W_{ij}(\mathbf{x}_{ij}, l_i)\cdot\left(\mathbf{v}_j - \mathbf{v}_i\right) \tag{10}$$

$$\text{with } \omega_i = -\sum_{j=1,N} V_j r_{ij}\frac{dW_i}{dr_{ij}} \tag{11}$$

$\omega_i$ being a correction factor that appears in variable smoothing length formulation.
It is also possible to rewrite the second and third equations of (5) in the SPH formalism, as

$$\frac{Du_i}{Dt} = -\sum_{j=1.N} V_j\left(g + \Pi_{ij}\right)\frac{\partial W_{ij}}{\partial x} + \frac{g}{d_{tot_i}}\sum_{j=1.N} V_j(h_j - h_i)\frac{\partial W_{ij}}{\partial x} - gS_{fx} \tag{12}$$

$$\frac{Dv_i}{Dt} = -\sum_{j=1.N} V_j\left(g + \Pi_{ij}\right)\frac{\partial W_{ij}}{\partial y} + \frac{g}{d_{tot_i}}\sum_{j=1.N} V_j(h_j - h_i)\frac{\partial W_{ij}}{\partial y} - gS_{fy} \tag{13}$$

$\Pi_{ij}$ being an artificial viscosity expressed as (*Monaghan*, 1992):

$$\begin{cases} \Pi_{ij} = -\alpha\dfrac{l_{ij}c_{ij}\left(\mathbf{v}_i - \mathbf{v}_j\right)\cdot\left(\mathbf{r}_i - \mathbf{r}_j\right)}{h_{ij}\left(\left|\mathbf{r}_i - \mathbf{r}_j\right|^2 + \varepsilon l_i^{\,2}\right)} & \left(\mathbf{v}_i - \mathbf{v}_j\right)\cdot\left(\mathbf{r}_i - \mathbf{r}_j\right) < 0 \\ \Pi_{ij} = 0 & \left(\mathbf{v}_i - \mathbf{v}_j\right)\cdot\left(\mathbf{r}_i - \mathbf{r}_j\right) \geq 0 \end{cases} \tag{14}$$

It is to be noted that the $\alpha$ parameter is here used to prevent particles collision, and is set at a value ($\alpha = 0.05$) which does not influence much the simulated waves mechanics. Moreover $l_{ij} = 0.5(l_i + l_j)$, $c_{ij} = 0.5(c_i + c_j)$, $h_{ij} = 0.5(h_i + h_j)$, $c_i = \sqrt{gd_{tot_i}}$ and $\varepsilon = 0.01$. At the generic time step $n$, the updated value of the smoothing length is evaluated using (6), while the total water

depth $d_{tot}$ is obtained solving the partial differential equation (10), such as

$$\begin{cases} d_{tot_{i,n+1}} = d_{tot_{i,n}} \, \mathrm{e}^{\lambda_n} \\ \lambda = \dfrac{dm\Delta t}{\omega_i}\left[ \sum_{j=1.N} V_j(\mathbf{v}_j - \mathbf{v}_i) \cdot \nabla W_{ij}\left(\mathbf{x}_{ij}, l_i\right) \right] \end{cases} \tag{15}$$

To evaluate $d_{tot}$ with a given accuracy, this equation is solved using a Newton-Raphson iterative process with the constraint to satisfy also eq (9). Finally, velocity and position are updated using a second order accurate and explicit Newmark scheme.

## 3 APPLICATIONS OF THE NSWE-SPH MODEL

Here we show the application of the implemented NSWE-SPH numerical model to simulate two benchmark test cases from laboratory studies. The first is a study on underwater landslide generated water waves performed by *Watts et al.* (2001). The second is a study on the force exerted on a tall structure by a dam break wave, realized by Yeh and Petroff at University of Washington and reported also in *Gómez-Gesteira and Dalrymple* (2004).

### 3.1 UNDERWATER LANDSLIDE GENERATED WATER WAVES

The experimental study described by *Watts et al.* (2001), consisted of water waves generated by the movement of an underwater landslide sliding down an inclined ramp. The underwater landslide was modelled as a solid body with a semi-elliptical shape, considering a 1:1000 scaling factor with a potential real landslide and the Froude similarity. Figure 2 presents a sketch of the physical model longitudinal section along with the parameters used to simulate the landslide movement which was expressed using the equation

$$s(t) = s_o \ln[\cos(t / t_o)]$$

with $s_o = u_t^2 / a_o$ and $t_o = u_t/a_o$. $a_o$ and $u_t$ denote the landslide initial acceleration and terminal velocity respectively, while the maximum movement of the underwater landslide is equal to $s_o$ and occurs in a total time $t_o$. Three different model configurations have been considered by *Watts et al.* (2001). Four wave gages were used to measure the water level oscillations, the first gage being located at $x = x_g$, i.e. just above the center of mass of the underwater landslide when it starts moving, and the others set 0.30 m apart (300 m in real scale) with increasing x.
The landslide effect has been simulated in the model modifying the continuity equation using the reduction factor of the landslide thickness, expressed as a function of the water depth as proposed by *Tinti et al.* (2005).
Test case c2 (see *Watts et al.*, 2001) is here considered, and presents a beach slope angle θ equal to 15° with the horizontal, the slide length b = 1.0 m, its thickness T = 0.0518 m, the starting depth of the slide d = 0.259 m, and the starting position $x_g$ = 1.166 m.

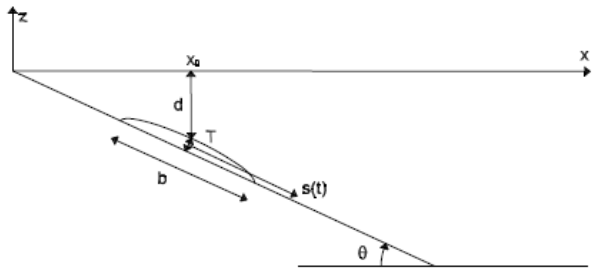


**Figure 2**. Physical model layout and notation

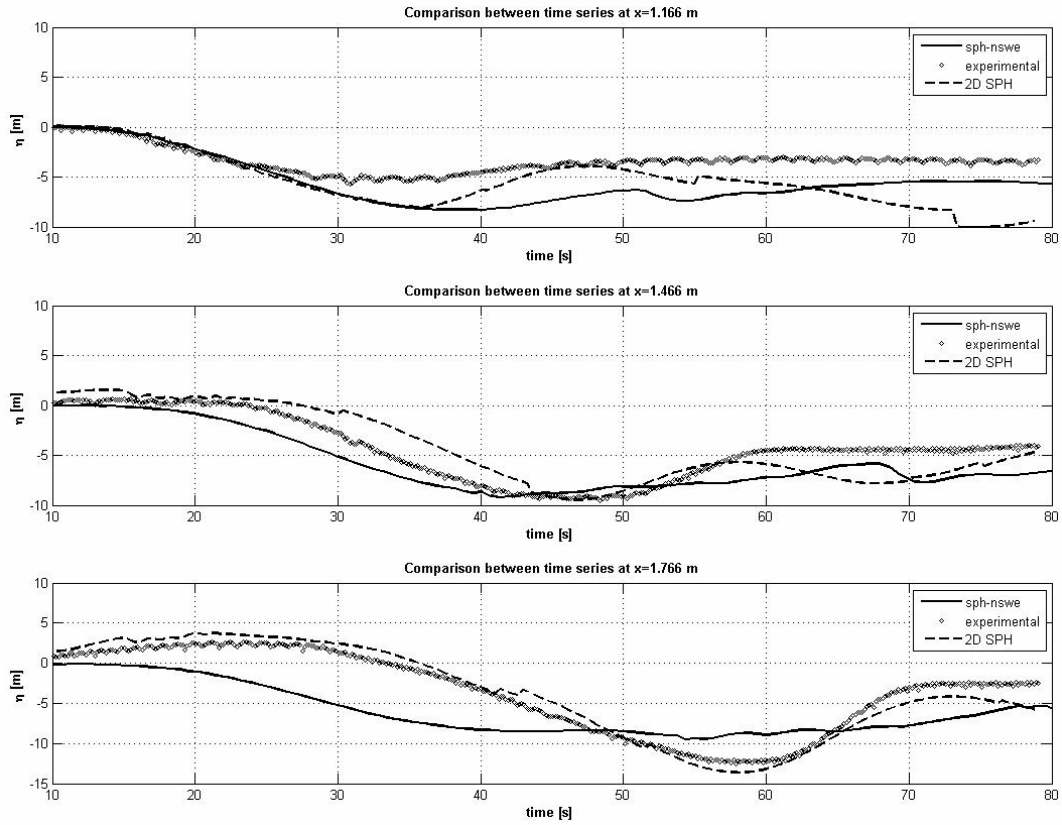


**Figure 3**. Comparison between time series of water surface elevation at tree different location, i.e. x=1.166m x=1.466m, x=1.766m (from top to bottom). The black line refers to NSWE-SPH simulation, dots refer to experimental data and the dashed line to 2DSPH numerical model (*Panizzo and Dalrymple*, 2004).

This 1D test was carried out using 238 particles with different volumes that remain constant during the simulation to ensure the same model resolution at different water depths. In fact working with the same volume for all the particles in the domain generate a very low numerical resolution in shallow waters. In our simulation we applied a quintic-spline kernel function, while a constant and small enough time step ($\Delta t$=0.0002 s) was chosen to respect the CFL condition.

The gradient kernel normalization proposed by *Bonet and Lok* (1999) was applied to assure the conservation of angular momentum, and to get a correct evaluation of the kernel gradient.

Plots in Figure 3 present the comparison between the time series of water surface elevation at x = 1.166 m, x = 1.466 m and x = 1.766 m from the shoreline at the left end of the wave flume, measured (dots), simulated with NSWE-SPH (continous line) and 2DSPH (*Panizzo and Dalrymple*, 2004). The agreement with experimental data is good considering both NSWE-SPH and 2D SPH results. About NSWE-SPH results, the fit is better at the first two wave gages, placed in shallower water with respect to the third wave gage. For the first signal, the NSWE-SPH model, as we expected, slightly overestimates the through being the turbulence very important near the shoreline and NSWE-SPH not able to correctly reproduce this phenomenon. The landslide potential energy, that in NSWE-SPH simulation is totally transferred to the fluid, is indeed partially dissipated by vortex formation and turbulence in the real case. Moreover for the last signal, the 2DSPH model is able of more accurately reproducing the water surface elevation than the NSWE-SPH model. Though, the carried out simulations demonstrate that it is possible to reproduce with good accuracy the problem at hand using a depth averaged model and saving as well a considerable amount of computation

time . It is worth to notice that NSWE-SPH runs in few minutes on a standard laptop computer, while classic 2DSPH runs in several hours.

### 3.2 DAM BREAK WAVE IMPACTING A TALL STRUCTURE

This test case has been carried out to evaluate the model capability to reproduce the wave force acting on a tall structure. Despite this phenomenon is fully 3D and has to be studied properly with fully 3D numerical models, such as VOF or 3DSPH, we want to show that also the NSWE-SPH numerical model is able to correctly simulate the wave impact on the structure with a good accuracy and with a low computation cost.

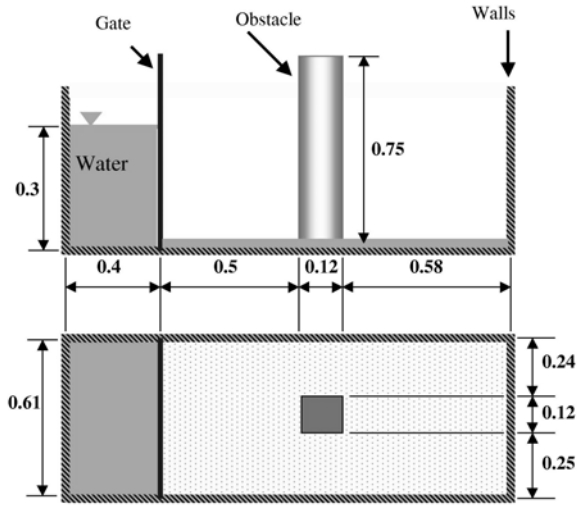


**Figure 4**. experimental setup, side and top view.

The reproduced experiment is a dam break problem confined in a rectangular box. The experimental box (Fig. 4) is 1.60 m long, 0.61 m wide and 0.75 m tall. The volume of water is initially kept at rest, with dimensions 0.40 m, 0.61 m, 0.3 m, by a very thin gate at one side of the box. The tall structure, which is 0.12 x 0.12 x 0.75 m, is placed 0.5 m downstream of the gate and 0.24 m far from the nearest tank side wall. Being experimentally very complicated to assure the absolute gate joints impermeability before the dam break, a initial layer of water (approximately 0.01 m) exists on the tank bottom. Experimental measurements included the time history of the fluid velocity at different locations as well as the net force acting on a structure. For more details about the experiment see *Gòmez-Gesteira et al.* (2004).

Previous numerical simulations of this case have been carried out by *Raad at Southern Methodist University (http://engr.smu.edu/waves/solid.html)* and by *Gòmez-Gesteira and Dalrymple* (2004). Here, the NSWE-SPH numerical simulation reproduces faithfully the experimental setup considering about 2624 particles with constant volume. The kernel function proposed by Gordon-Johnson has been applied. A constant and small enough time step ($\Delta t$=0.01 s) was chosen to respect the CFL condition. The four basin walls have been simulated using ghost particles. Figure 5 shows four snapshots of the dam break wave evolution in the numerical wave tank.

Figure 6 shows the comparison between numerical (dots) and experimental data (solid line) of the force on the structure and the water velocity in front of the obstacle. The picture shows a good agreement between simulated and observed force time series, while a slight time lag exists between numerical and measured data of the water velocity. This time shift could be related to the uncertainty about the instant of start up measurement in the experimental apparatus. These results show that NSWE-SPH is satisfactory able to deal with water waves

interacting with structures.
It is worth stressing that a new and promising method has been used to calculate the force applied by the water on the structure. The solid boundaries of the structure have been reproduced using repulsive forces in the momentum equation to impose the boundary conditions. In other words each particle that represents the structure induces forces on the water particles avoiding penetration and guaranteeing perfect wave reflection and impermeability. By summing up the repulsive force exchanged by all the particles representing the structure and the fluid, the total force exchanged by the structure itself and the water is obtained. This corresponds to the force applied by the water.

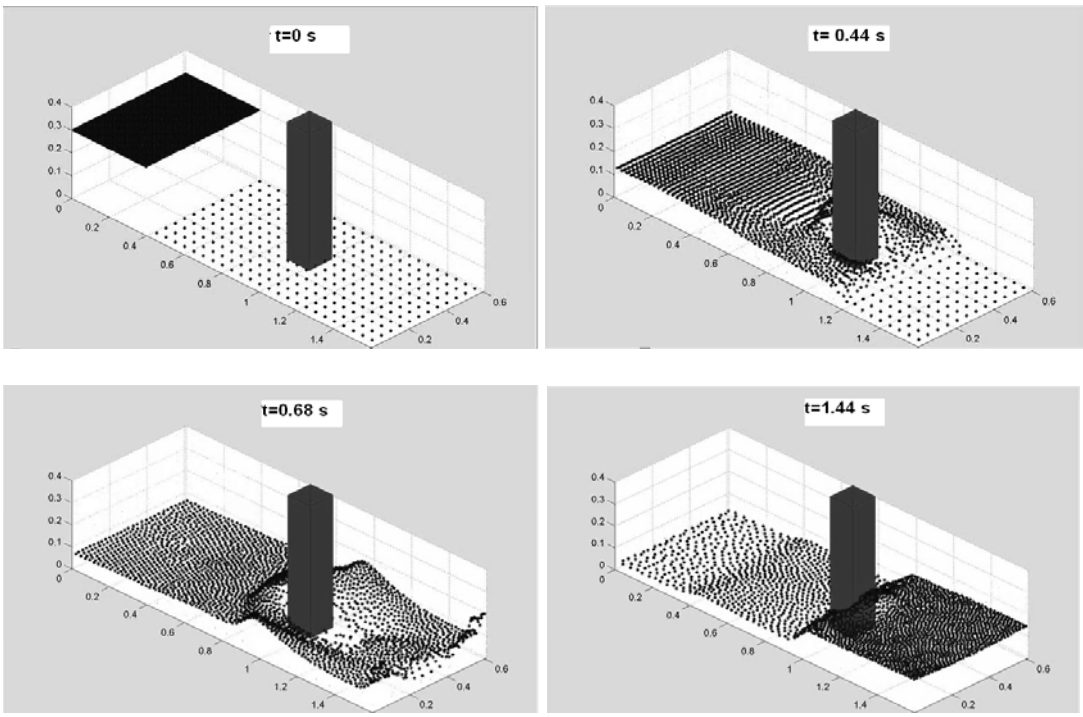


**Figure 5.** Dam break evolution, t=0.0 s initial configuration; t=0.44 s wave hitting the structure; t=0.68 s wave passing the structure; t=1.44 s reflected wave hitting back the structure

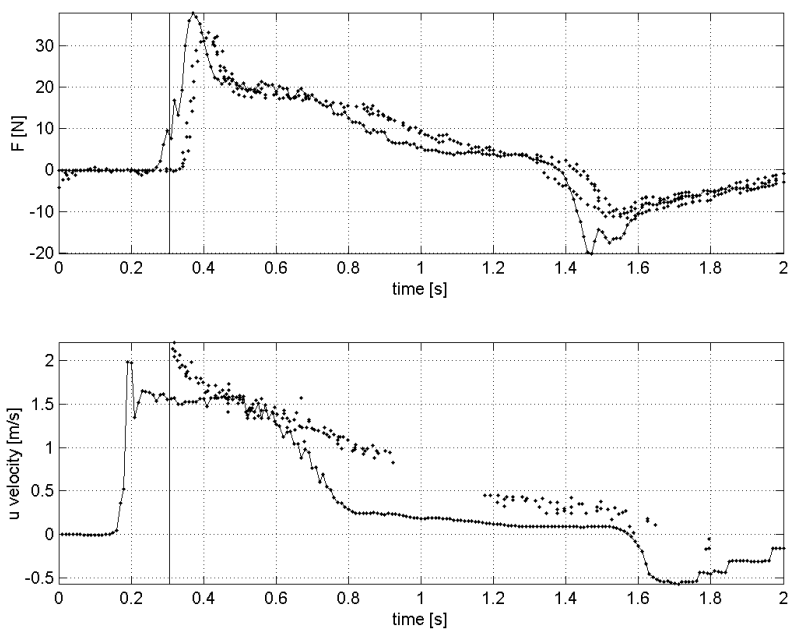


**Figure 6.** Comparison between NSWE-SPH numerical results (continuous line) and experimental data (dots)

## 4 CONCLUSIONS AND ONGOING RESEARCHES

A numerical model based on Nonlinear Shallow Water Equations (NSWE) solved by means of the Smoothed Particle Hydrodynamics (SPH) approach has been presented. The numerical model has been developed with the aim of simulating the generation and the propagation of tsunamis waves and their run-up at the coast, and it has been implemented within the framework of a research program on tsunamis waves currently carried out at the LIAM laboratory of L'Aquila University (for more details see www.tsunamis.it).

Indeed, the classic set of NSWE equations has been rewritten using the SPH formalism. Details on the implemented numerical model have been given and the model mathematics explained. The model applications to physical model tests dealing with the main theme of fluid structure interaction have been presented.

Despite these phenomena are three-dimensional and may be properly studied with fully 3D numerical models such as the VOF or 3DSPH, we showed that accepting some small approximations, also a 2D Lagrangian and depth integrated numerical model is able to correctly simulating wave impacts on structures with a good accuracy and with the main advantage related to very low computational costs.

The next step will be the simulation of 3D landslide generated tsunamis comparing the simulation results with the data collected during a new experiment carried out at the LIAM laboratory of L'Aquila University (*Bellotti et al.*, 2006).

## AKNOWLEDGMENTS

This work has been funded by MIUR-PRIN "Onde di maremoto generate da frane in corpi idrici: meccanica della generazione e della propagazione, sviluppo di modelli previsionali e di sistemi di allerta in tempo reale basati su misure mareografiche", and by Italian Government, Department of Protezione Civile (contract "Ricostruzione del campo di moto ondoso generato a Stromboli dall'evento di frana del 30/12/2002 e previsione del campo d'onda in relazione a scenari alternativi di frana").

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