SPH development at Cranfield University

Prof. Rade Vignjevic

IV SPHERIC
Nantes, May 2009

SPH development at Cranfield University

Cranfield UNIVERSITY

Presentation outline

1. Introduction (CU, Motivation)
2. Normalised Corrected SPH
3. Non-collocational SPH
4. Contact algorithm, FE – SPH coupling
5. Damage modelling

Cranfield University: History

Formed in 1946 as **College of Aeronautics** (centre of excellence in aerospace)

Postgraduate University 1992

Centre of Applied Research

Five Major Schools

- Health
- Management
- **Engineering**
- Applied Science

Cranfield University:

- Entirely Post Graduate, 35% PhD
- MSc Courses of 45 Week Duration
- International Industrial & Government Funded Research
- The UK National Flying Laboratory
- Applied approach to teaching and research
- Strong Industrial Links
- Worldwide Reputation
- £240 Million Turnover - 60% From Research

Motivation for our SPH work

Cranfield UNIVERSITY

Challenging problems of computational mechanics are often characterised by:

- Extremely large deformations
- Tracking of interfaces between solids and liquid/gas
- Propagation of cracks with arbitrary and complex paths
- Change of boundaries of material that has undergone extensive micro cracking or phase change and failure.

Motivation for the SPH work

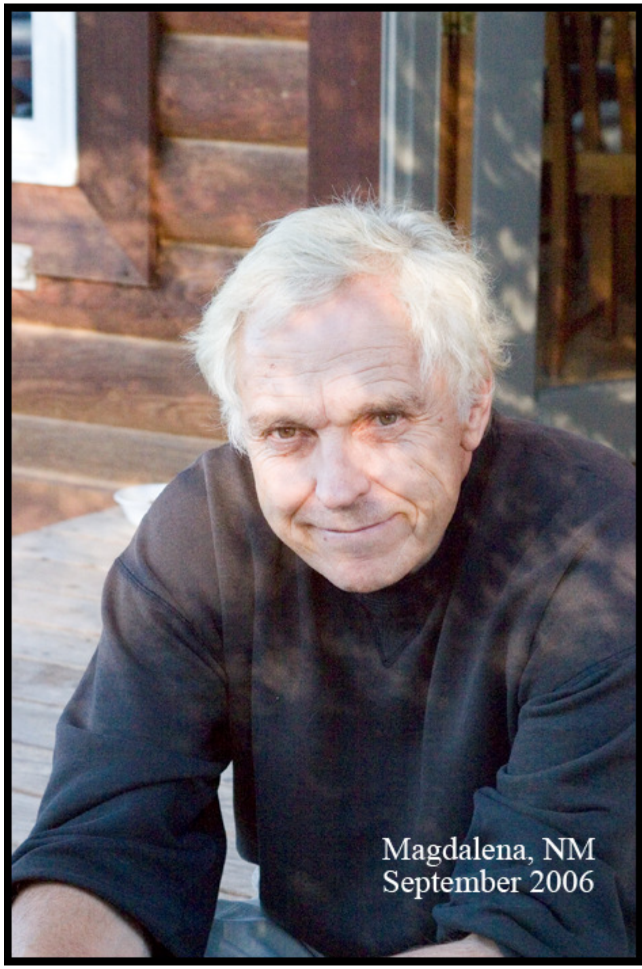
Cranfield UNIVERSITY

Problem solving

Manufacturing:
Extrusion, Moulding, Friction Stir Welding, Manufacture of composites, …

Safety and Crashworthiness:
Hypervelocity impacts on spacecraft, Aircraft impacts on water/soft soil, Bird strike ...

Defence:
Armour penetration (metal, ceramics), Warhead fragmentation, Shape charges…



Cranfield UNIVERSITY

People that inspired and influenced SPH research at Cranfield:

L. Libersky

J. J. Monaghan

Corrected Normalised SPH (CNSPH)

R. Vignjevic, J. Campbell , L. Libersky; *A treatment of zero-energy modes in the smoothed particle hydrodynamics method,* Comput. Methods Appl. Mech. Engrg. 184 (2000), pp. 67-85, (Received 28 October 1998)

R. Vignjevic, J. Reveles, J. Campbell; *SPH in a Total Lagrangian Formalism*, Computer Methods in Engineering and Science, vol.14, no.3, pp.181-198, 2006

Corrected Normalised SPH
CNSPH

Cranfield UNIVERSITY

Emmy Noether, "Invariante Variationsprobleme,
" Nachr. v. d. Ges. d. Wiss. zu Göttingen 1918, pp. 235-257

Specifically: The invariance with respect to translational and rotational transformations

Corrected Normalised SPH CNSPH

Space homogeneity

$$\langle \mathbf{x} \rangle = \sum_J \frac{m_J}{\rho_J} \mathbf{x}_J W\left(\left|\mathbf{x}_I - \mathbf{x}_J\right|, h\right)$$

$\langle \mathbf{x} \rangle$ is interpolated solution space

$$\langle \mathbf{x}' \rangle\Big|_{\mathbf{x}'=\mathbf{x}'_I} = \sum_J \frac{m_J}{\rho_J} \mathbf{x}'_J W\left(\left|\mathbf{x}'_I - \mathbf{x}'_J\right|, h\right)$$

$$\mathbf{x}' = \mathbf{x} - \Delta\mathbf{x}$$

$$\langle \mathbf{x}' \rangle = \langle \mathbf{x} \rangle - \Delta\mathbf{x} \sum_J \frac{m_J}{\rho_J} W\left(\left|\mathbf{x}_I - \mathbf{x}_J\right|, h\right)$$

$$\boxed{\sum_J \frac{m_J}{\rho_J} W\left(\left|\mathbf{x}_I - \mathbf{x}_J\right|, h\right) = 1}$$

Corrected Normalised SPH
CNSPH

Cranfield UNIVERSITY

Space isotropy

$$\langle \mathbf{x} \rangle = \sum_J \frac{m_J}{\rho_J} \mathbf{x}_J W \left(\left| \mathbf{x}_I - \mathbf{x}_J \right|, h \right)$$

$\langle \mathbf{x} \rangle$ is interpolated solution space

$$\langle \mathbf{x}' \rangle \Big|_{\mathbf{x}'=\mathbf{x}'_I} = \sum_J \frac{m_J}{\rho_J} \mathbf{x}'_J W \left(\left| \mathbf{x}'_I - \mathbf{x}'_J \right|, h \right)$$

$$\mathbf{x}' = \mathbf{C} \cdot \mathbf{x}$$

$$\langle \mathbf{x}' \rangle \equiv \langle \mathbf{C} \cdot \mathbf{x} \rangle = \langle \mathbf{C} \rangle \cdot \langle \mathbf{x} \rangle = \mathbf{C} \cdot \langle \mathbf{x} \rangle$$

$$\langle \mathbf{C} \rangle = \mathbf{C}$$

$$\sum_{J=1}^{nnbr} \frac{m_J}{\rho_J} \mathbf{x}_J \otimes \nabla W(\mathbf{x}_I - \mathbf{x}_J, h) = \mathbf{I}$$

Corrected Normalised SPH CNSPH

Interpolation should not violate the following properties of space:

	Homogeneity	Anisotropy
Condit.	$\sum_{j=1}^{nnbr} \frac{m_j}{\rho_j} W(x_i - x_j, h) = 1$	$\sum_{j=1}^{nnbr} \frac{m_j}{\rho_j} \mathbf{x}_j \otimes \nabla W(x_i - x_j, h) = \mathbf{1}$
Correct.	$\tilde{W}_{ij} = \frac{W(x_i - x_j, h)}{\sum_{j=1}^{nnbr} \frac{m_j}{\rho_j} W(x_i - x_j, h)}$	$\tilde{\nabla} \tilde{\mathbf{W}}_{ij} = \nabla \tilde{\mathbf{W}}_{ij} \left(\sum_{j=1}^{nnbr} \frac{m_j}{\rho_j} x_j \otimes \nabla \tilde{\mathbf{W}}_{ij} \right)^{-1}$

CNSPH Form of Governing Equations

Cranfield UNIVERSITY

Conservation of:

Mass

$$\left\langle \dot{\rho}_i \right\rangle = \rho_i \sum_{j=1}^{nnbr} \frac{m_j}{\rho_j} \left(\mathbf{v}_j - \mathbf{v}_i \right) \cdot \tilde{\nabla} \tilde{\mathbf{W}}_{ij}$$

Momentum

$$\left\langle \dot{\mathbf{v}}_{\mathbf{i}} \right\rangle = \sum_{j=1}^{nnbr} m_j \left(\frac{\boldsymbol{\sigma}_i}{\rho_i^2} + \frac{\boldsymbol{\sigma}_j}{\rho_j^2} \right) \cdot \tilde{\nabla} \tilde{\mathbf{W}}_{ij}$$

Energy

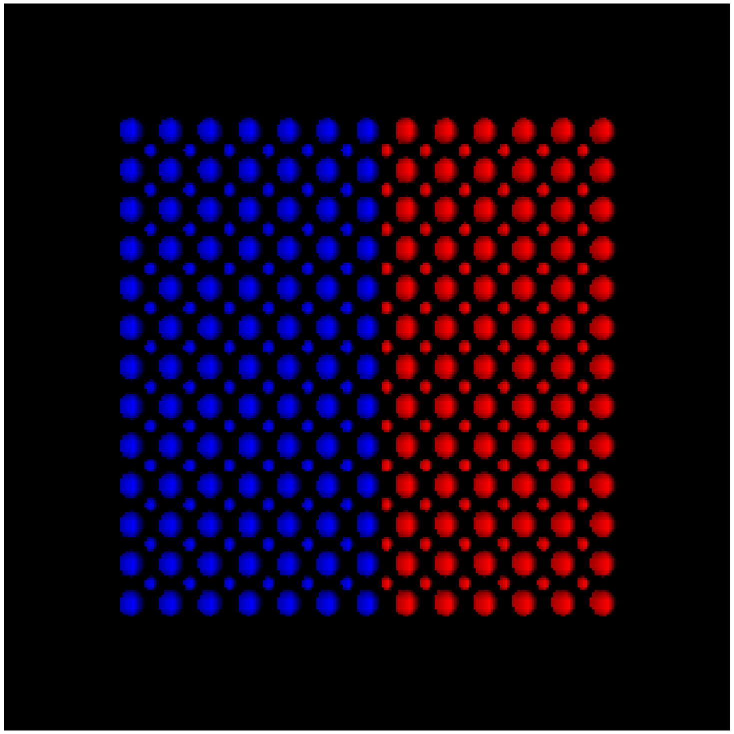
$$\left\langle \dot{e} \right\rangle = -\frac{\boldsymbol{\sigma}_i}{\rho_i^2} \sum_{j=1}^{nnbr} m_j \left(\mathbf{v}_j - \mathbf{v}_i \right) \cdot \tilde{\nabla} \tilde{\mathbf{W}}_{ij}$$

Where:

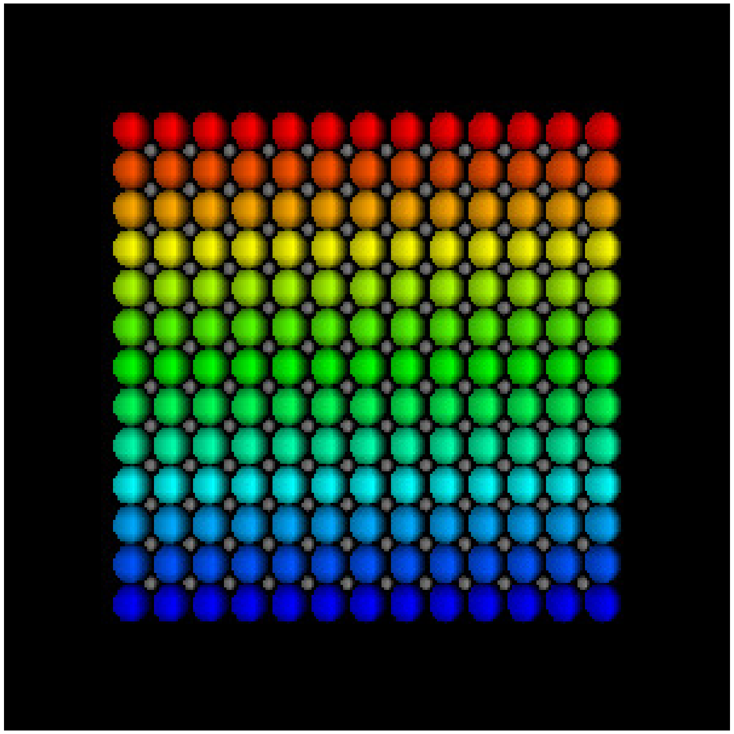
$$\tilde{\mathbf{W}}_{ij} = \mathbf{W}_{ij} \left(\sum_{j=1}^{nnbr} \frac{m_j}{\rho_j} \left(x_i - x_j \right) \mathbf{W}_{ij} \right)^{-1}, \quad \tilde{\nabla} \tilde{\mathbf{W}}_{ij} = \nabla \tilde{\mathbf{W}}_{ij} \cdot \left(\sum_{j=1}^{nnbr} \frac{m_j}{\rho_j} \left(x_i - x_j \right) \otimes \nabla \tilde{\mathbf{W}}_{ij} \right)^{-1}$$

CNSPH Conservation of Momentum and A-momentum

Cranfield UNIVERSITY



Materials



X -Velocity

Corrected Normalised SPH Summary

- First order consistency
- Conservation of linear and angular momentum
- Homogeneity and isotropy of space maintained in the SPH discretisation

SPH in a Total Lagrangian Formalism

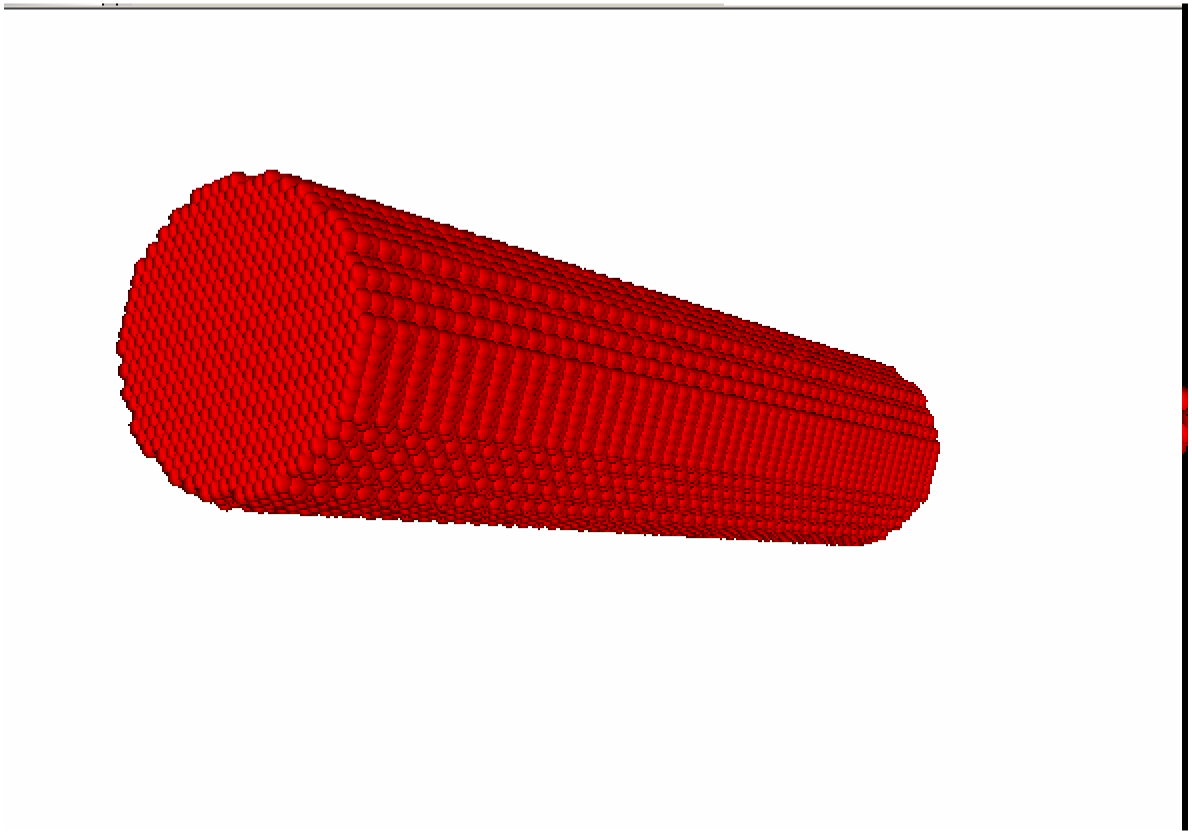
Cranfield UNIVERSITY

R. Vignjevic, J. Reveles, J. Campbell; SPH in a Total Lagrangian Formalism, Computer Methods in Engineering and Science, Vol.14, No.3, pp.181-198, 2006

SPH in a Total Lagrangian Formalism

Cranfield UNIVERSITY

Taylor test for
OFHPC copper
180 m/s



SPH in a Total Lagrangian Formalism

Cranfield UNIVERSITY

The mapping from material into spatial coordinates is

$\mathbf{x} = \phi(\mathbf{X}, t)$.

The displacement of a material point is given by the difference between its current position and its original position.

$$\mathbf{u}(X, t) = \phi(\mathbf{X}, t) - \phi(\mathbf{X}, 0) = \phi(\mathbf{X}, t) - \mathbf{X} = \mathbf{x} - \mathbf{X}$$

SPH in a Total Lagrangian Formalism

The deformation gradient **F**

$$\mathbf{F} = \frac{\partial \mathbf{x}}{\partial \mathbf{X}} = \frac{\partial(\mathbf{u}+\mathbf{X})}{\partial \mathbf{X}} = \frac{\partial \mathbf{u}}{\partial \mathbf{X}} + \mathbf{I}$$

Discretised form

$$\langle \mathbf{F}_I \rangle = \left(-\sum_{J \in S} (\mathbf{u_J} - \mathbf{u_I}) \otimes \nabla_{\mathbf{0}} \tilde{W}_{\mathbf{IJ}} \mathbf{V}_{\mathbf{J}}^{\mathbf{0}} \right) . \mathbf{B} + \mathbf{I} \quad \mathbf{B} = \left(-\sum_{j \in S} \frac{m_j}{\rho_j} (\mathbf{X}_j - \mathbf{X}_i) \otimes \nabla \tilde{W} \right)^{-1}$$

Velocity gradient

$$\langle \dot{\mathbf{F}}_I \rangle = -\sum_{J \in S} (\mathbf{v}_J - \mathbf{v}_I) \otimes \nabla_{\mathbf{0}} \tilde{W}_{IJ} \mathbf{V}_J^{\mathbf{0}}$$

Cranfield UNIVERSITY

SPH in a Total Lagrangian Formalism

Cranfield UNIVERSITY

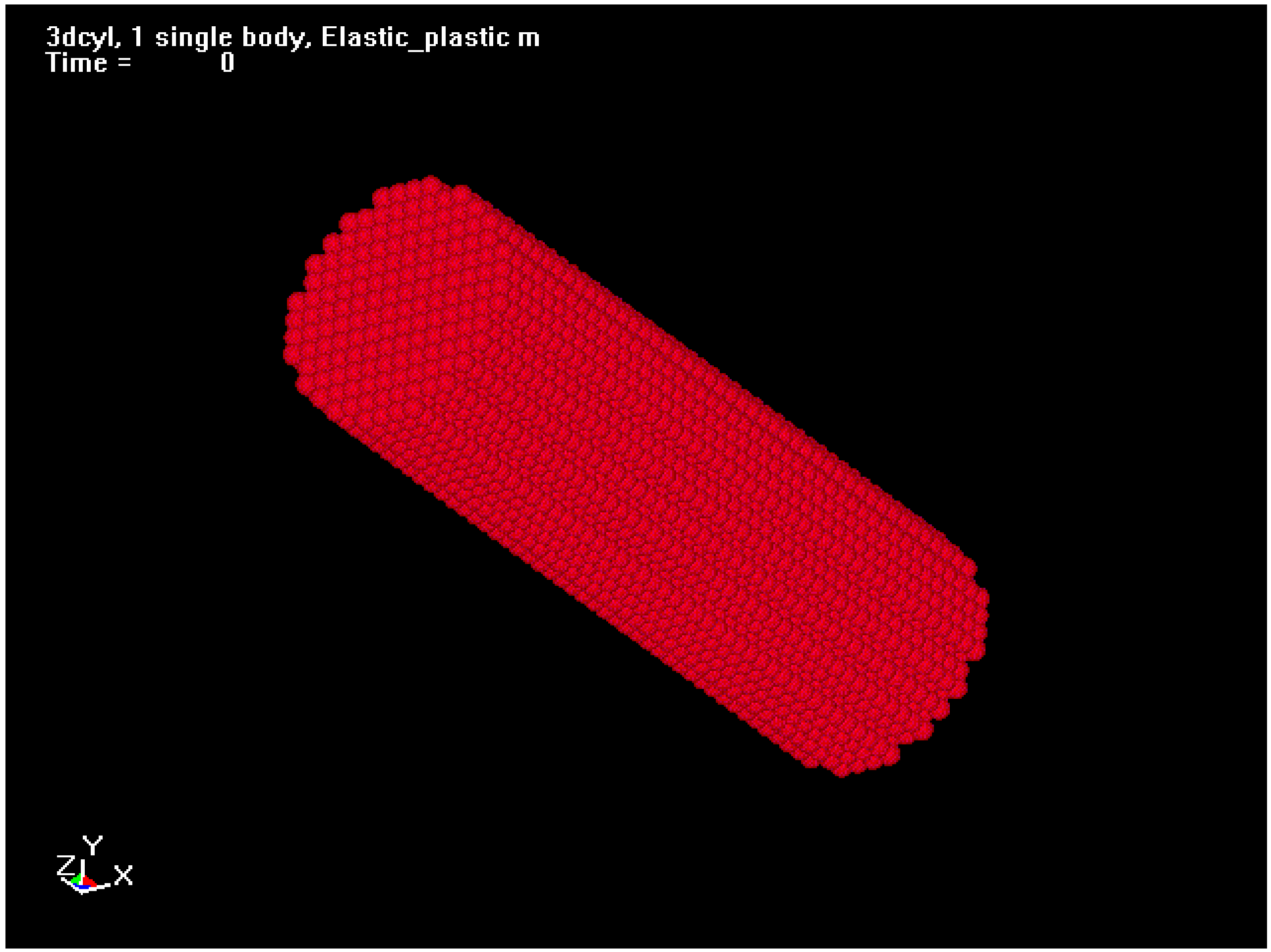
Conservation Equations in the Total Lagrangian formalism

	Continuous	Discretised
Mass	$\rho = J^{-1}\rho_0$	$\rho = \langle \mathbf{F}_I \rangle^{-1} \rho_0$
Momentum	$\ddot{\mathbf{u}} = \frac{\mathbf{1}}{\rho_0}\nabla_{\mathbf{0}}\mathbf{P} + \mathbf{b}$	$\langle \mathbf{a}_I \rangle = \left(-\sum_{J \in S} (\mathbf{P_J} - \mathbf{P_I}) \otimes \nabla_{\mathbf{0}} \tilde{W}_{\mathbf{IJ}} \mathbf{V_J^0} \right) : \mathbf{B}$
Energy	$\dot{e} = \frac{\mathbf{1}}{\rho_0}\nabla_{\mathbf{0}}\dot{\mathbf{F}} : \mathbf{P}$	$\langle \dot{e}_I \rangle = \mathbf{P}_J : \left[\left(-\sum_{J \in S} \frac{m_J}{\rho_I \rho_J} (\mathbf{v}_J - \mathbf{v}_I) \otimes \nabla_0 \tilde{W}_{IJ} \mathbf{V}_J^0 \right) . \mathbf{B} \right]$

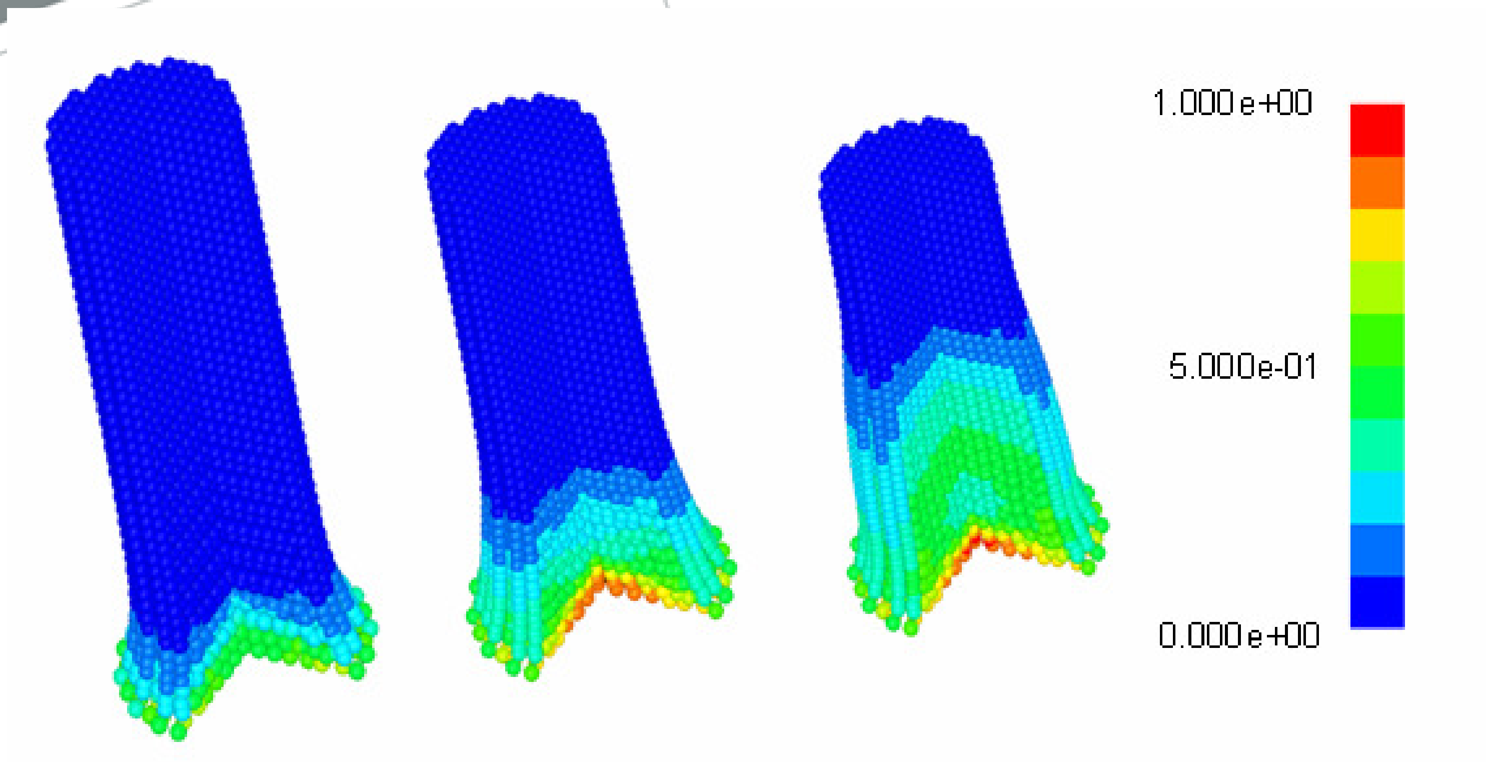
SPH in a Total Lagrangian Formalism

Cranfield UNIVERSITY

Taylor test for OFHPC copper 180 m/s



SPH in a Total Lagrangian Formalism

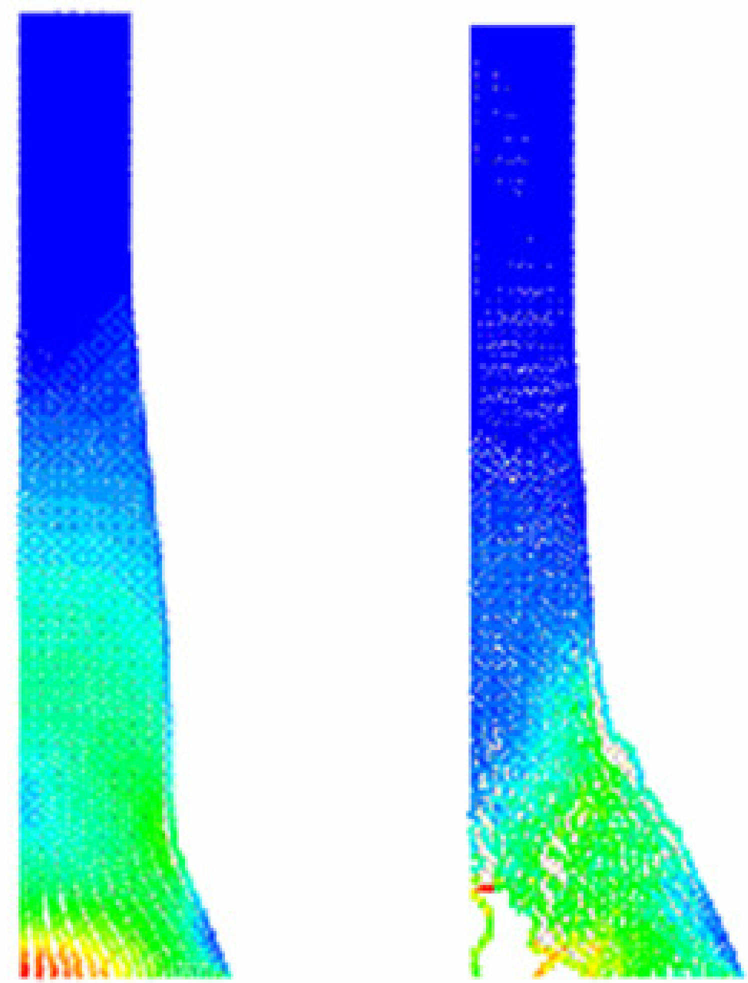


3-D Taylor test, effective plastic strain

SPH in a Total Lagrangian Formalism

Cranfield UNIVERSITY

Total Lagrangian SPH vs. conventional SPH



SPH in a Total Lagrangian Formalism Summary

- Stable well behaved
- Applicable to finite deformations
- Combined with Eulerian SPH when modelling extremely large deformations and failure

Non-Collocational SPH

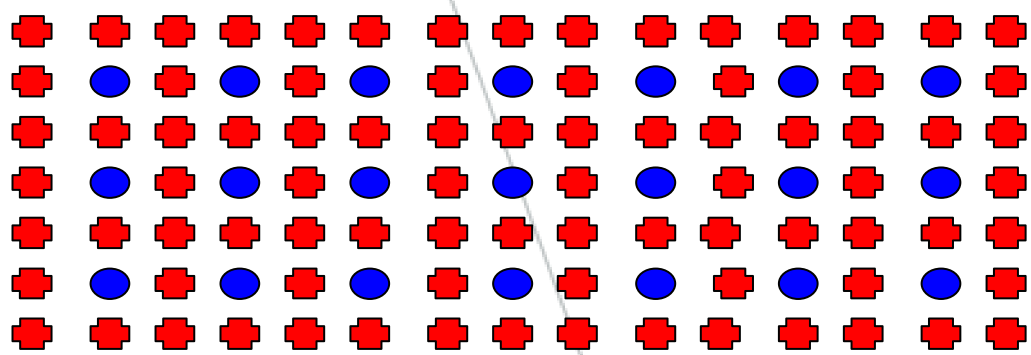
R. Vignjevic, J. Campbell , L. Libersky; *A treatment of zero-energy modes in the smoothed particle hydrodynamics method,* Comput. Methods Appl. Mech. Engrg. 184 (2000), pp. 67-85, (Received 28 October 1998)

Non-Collocational SPH

Two types of particles:

Velocity

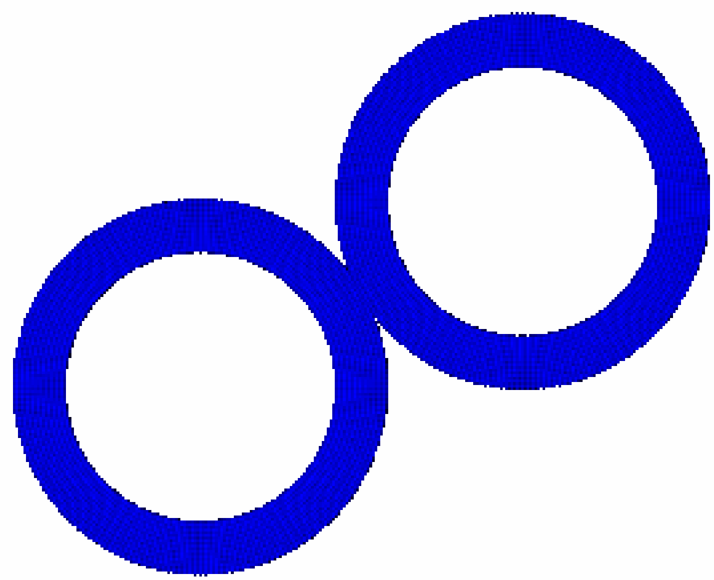
Stress



Easy to apply essential boundary conditions (similar to FE)

Non-Collocational SPH

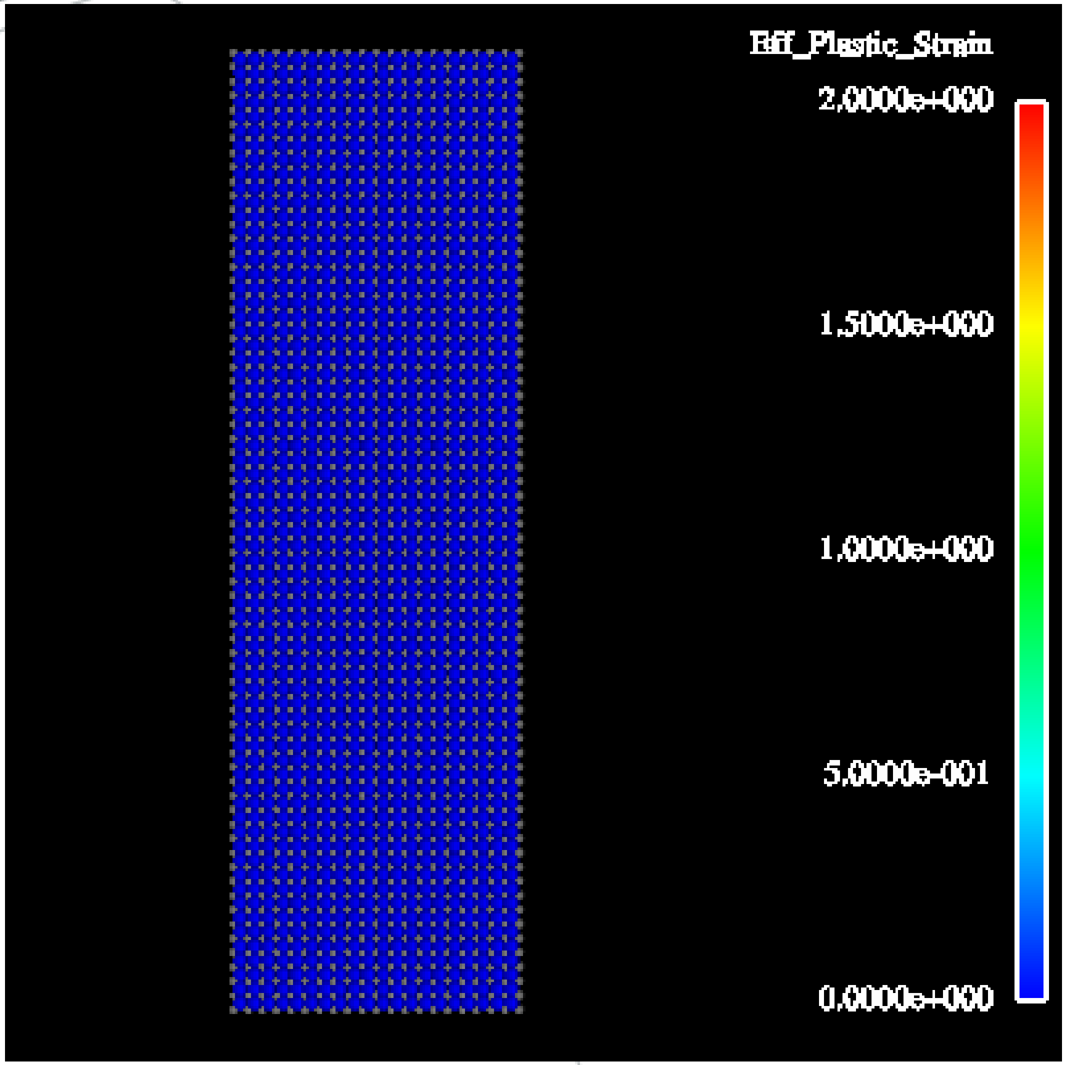
Collision of elastic hoops



Non-Collocational SPH

Cranfield UNIVERSITY

Taylor test for
OFHPC copper
180 m/s



Non-Collocational SPH Summary

Cranfield UNIVERSITY

- Easy to apply to 1D and 2D structural elements, especially advantageous when modelling failure (ongoing work)
- Increased difficulties to extend to 3D continuum (update of stress particle locations)

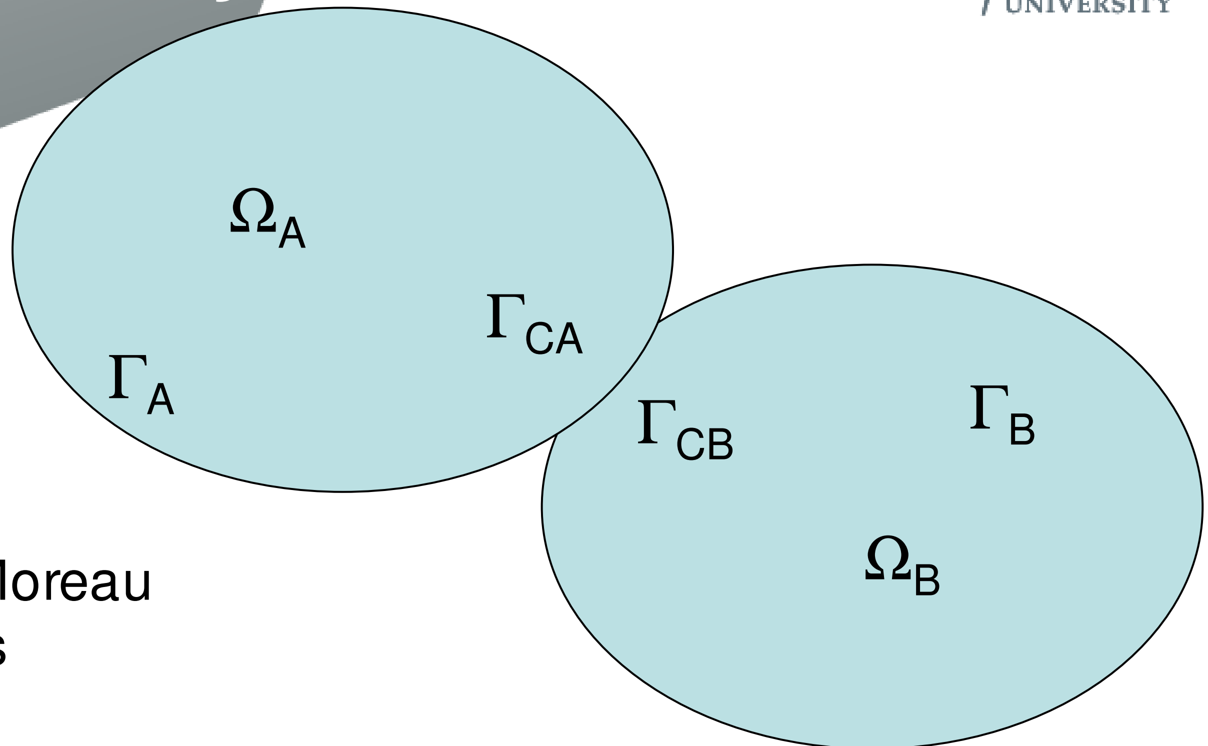
Contact Algorithm

R. Vignjevic; T. De Vuyst; and J. Campbell; *A Frictionless Contact Algorithm for Meshless Methods* , CMES, Vol. 13, No. 1, pp. 35-48, 2006

T. De Vuyst, R. Vignjevic and J. C. Campbell; *Modelling of Fluid-Structure Impact Problems using a Coupled SPH-FE solver*, Journal of Impact Engineering, Vol. 31, No. 8, pp. 1054-1064, 2005

J. Campbell, R. Vignjevic, L. Libersky; *A Contact Algorithm for Smoothed Particle Hydrodynamics*, Computer Methods in Applied Mechanics and Engineering, Vol. 184, No. 1, pp. 49-65, 2000

Contact Initial Boundary Value Problem



Traction on the contact surface

$$t_n = \hat{\mathbf{n}}_{\Gamma_{CB}} \cdot \boldsymbol{\sigma}$$

The Hertz – Signorini – Moreau (Kuhn-Tucker) Conditions for frictionless contact:

$$g \le 0\,, \quad t_{\mathbf{n}} \ge 0\,, \quad g\, t_{\mathbf{n}} = 0$$

$$g = \hat{\mathbf{n}}_{\Gamma_{CB}} \cdot \left(\mathbf{x}_B - \mathbf{x}_A\right)$$

In the variational rate form the contact constraint imposed by:

$$\delta G = \delta G\left(t_n \cdot \dot{g}\right) = 0$$

For a body in contact $\dot{g} = 0$

Contact Initial Boundary Value Problem

Cranfield UNIVERSITY

$\nabla \cdot \boldsymbol{\sigma} + \rho \mathbf{b} = \rho \mathbf{a}$ on

$\Omega = \Omega_A \cup \Omega_B$

$\boldsymbol{\sigma} \cdot \mathbf{n} = \bar{\mathbf{t}}$ on Γ_t,

$\mathbf{u} = \bar{\mathbf{u}}$ on Γ_u

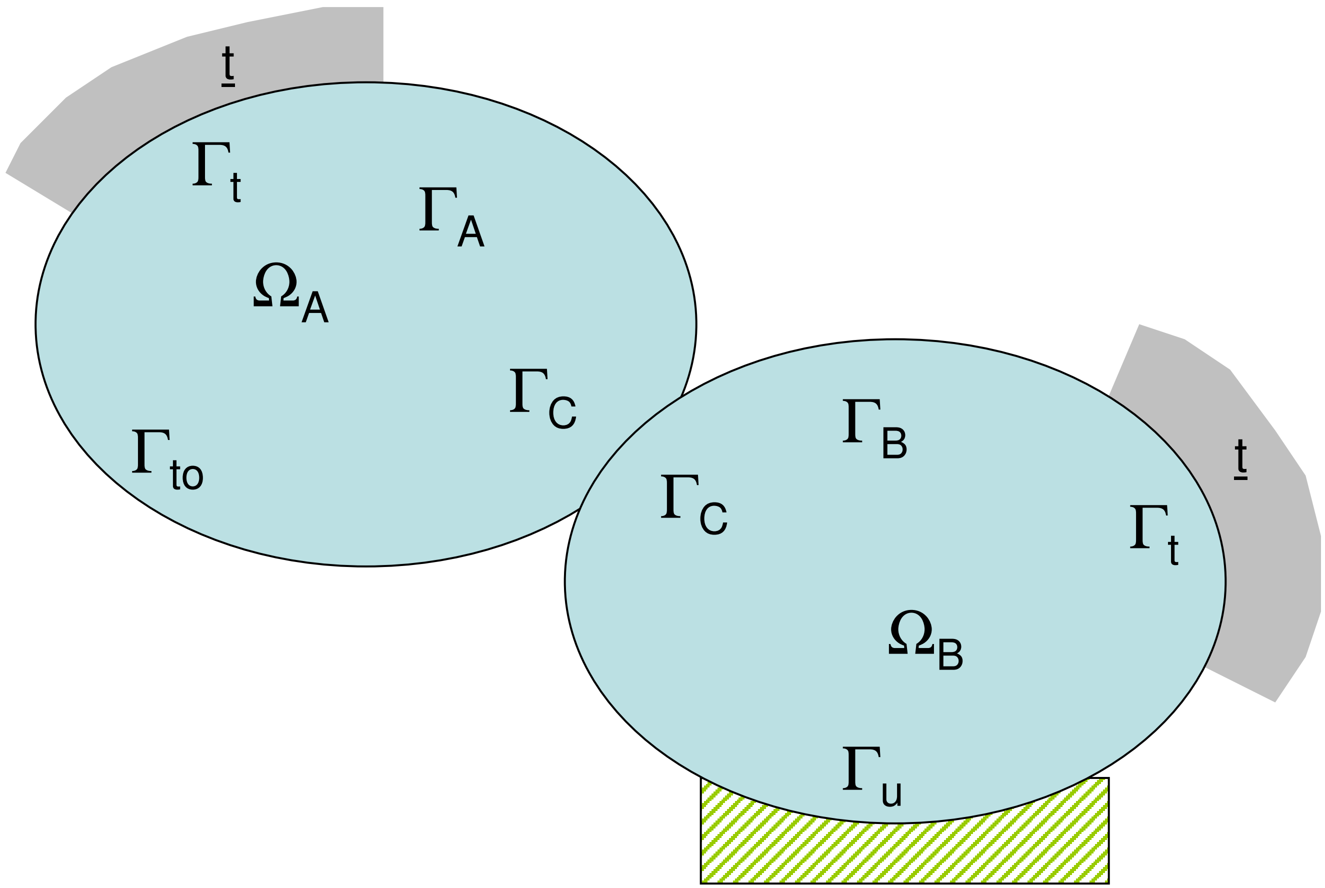
$\bar{\mathbf{t}}_A = -\bar{\mathbf{t}}_B$ on

$\Gamma_C = \Gamma_A \cap \Gamma_B$

$\mathbf{t}_N \geq 0$

$g \leq 0$

$\mathbf{t}_N \cdot g = 0$



Where $\Gamma_t \cup \Gamma_{to} \cup \Gamma_u = \Gamma$, and
$\Gamma_t \cap \Gamma_{to} = \varnothing$, $\Gamma_{to} \cap \Gamma_u = \varnothing$, $\Gamma_t \cap \Gamma_u = \varnothing$.

Discretised Contact Initi
Boundary Value Problem

Cranfield UNIVERSITY

$$\int_{\Gamma_t} w\boldsymbol{\sigma}\cdot n \, d\Gamma - \int_{\Omega} \nabla w \cdot \boldsymbol{\sigma} \, dV = \int_{\Omega} w\rho(\ddot{\mathbf{d}} - \mathbf{b}) \, dV - \int_{\Gamma_c} w\,\bar{\mathbf{t}} d\Gamma$$

A weak form of the initial boundary value problem with contact.

This equation discretised in space:

$$\int_{\Omega} \rho \mathbf{N}^T \mathbf{N} \, dV \ddot{\mathbf{d}} + \int_{\Omega} \mathbf{B}^T \boldsymbol{\sigma} \, dV \mathbf{d} - \int_{\Omega} \mathbf{N}^T \mathbf{b} \, dV - \boxed{\int_{\Omega_{CBL}} \mathbf{N}^T \mathbf{b}_{\mathbf{c}} \, dV} -$$

$$- \int_{\Gamma_t \cup \Gamma_u} \left(\mathbf{N}^T \boldsymbol{\sigma}\right) \cdot \mathbf{n} \, d\Gamma = 0$$

Where: $w = \mathbf{Nd}$ and **N** is a shape functions matrix

Discretised Contact Initia
Boundary Value Problem

In the SPH method **N** has the following form:

$$N_{ij} = \frac{m_j}{\rho_j} \frac{W_j(x_i)}{\sum_{j=1}^{np} W_j(x_i)}$$

Where:

I - particle at which shape function is evaluated,
J - particle at which the shape function is centered,
Np - number of neighbors for the *i* particle,
d - nodal displacement vector,
W - SPH kernel function (B-spline)

Contact Force

Cranfield UNIVERSITY

Due to the diffused nature of boundary in the conventional SPH the contact force is defined as:

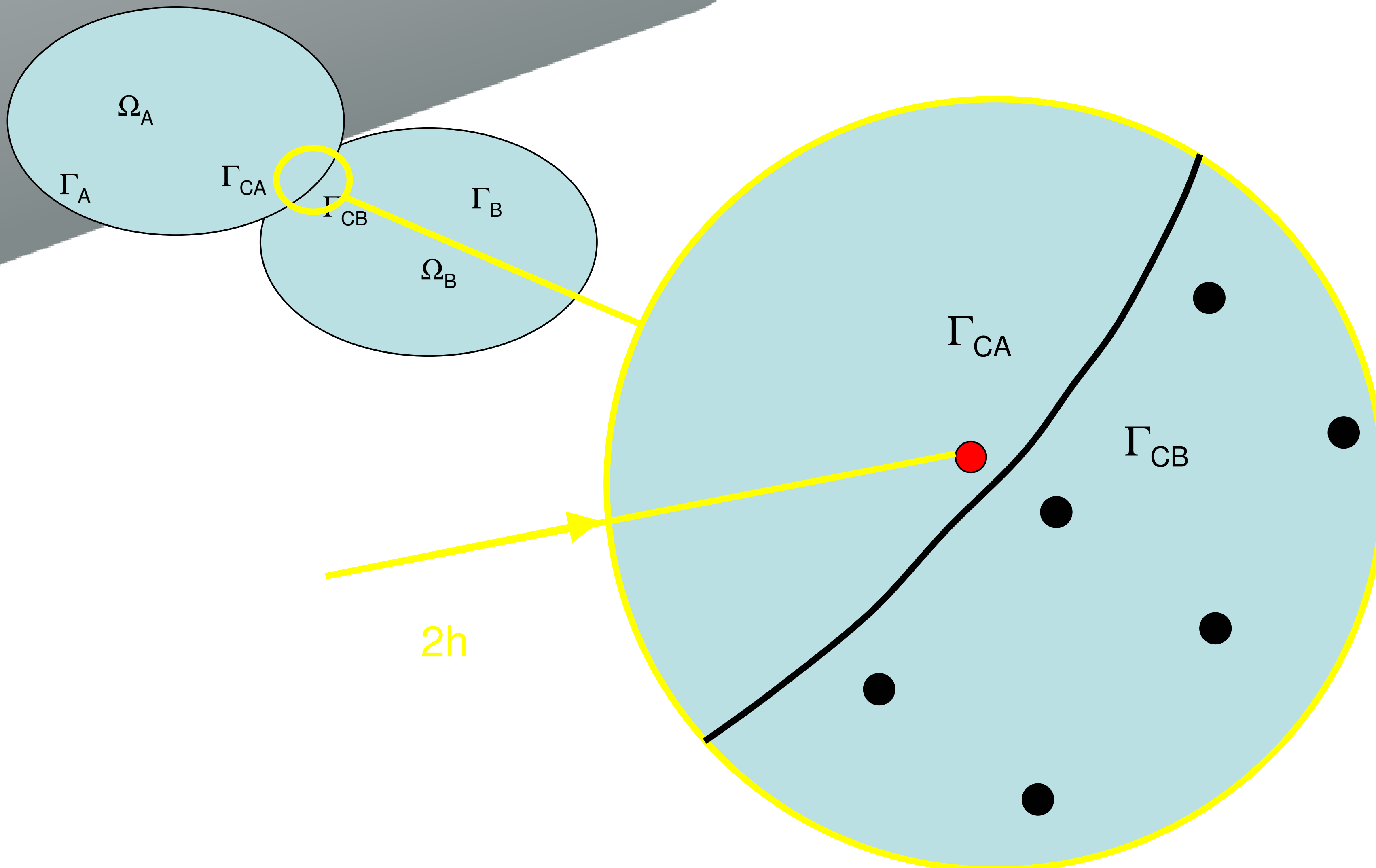
$$\mathbf{f_c} = \int_{\Omega_{CBL}} \mathbf{N}^T \mathbf{b_c} dV$$

Where specific contact force is defined as gradient of contact potential:

$$\phi(x_A) = \int_{\Omega_{CBL}} K\left(\frac{W(x_A - x_B)}{W(\Delta p_{avg})}\right)^n dV \qquad \phi(x_i) = \sum_j^{NCONT} \frac{m_j}{\rho_j} K\left(\frac{W(r_{ij})}{W(\Delta p_{avg})}\right)^n.$$

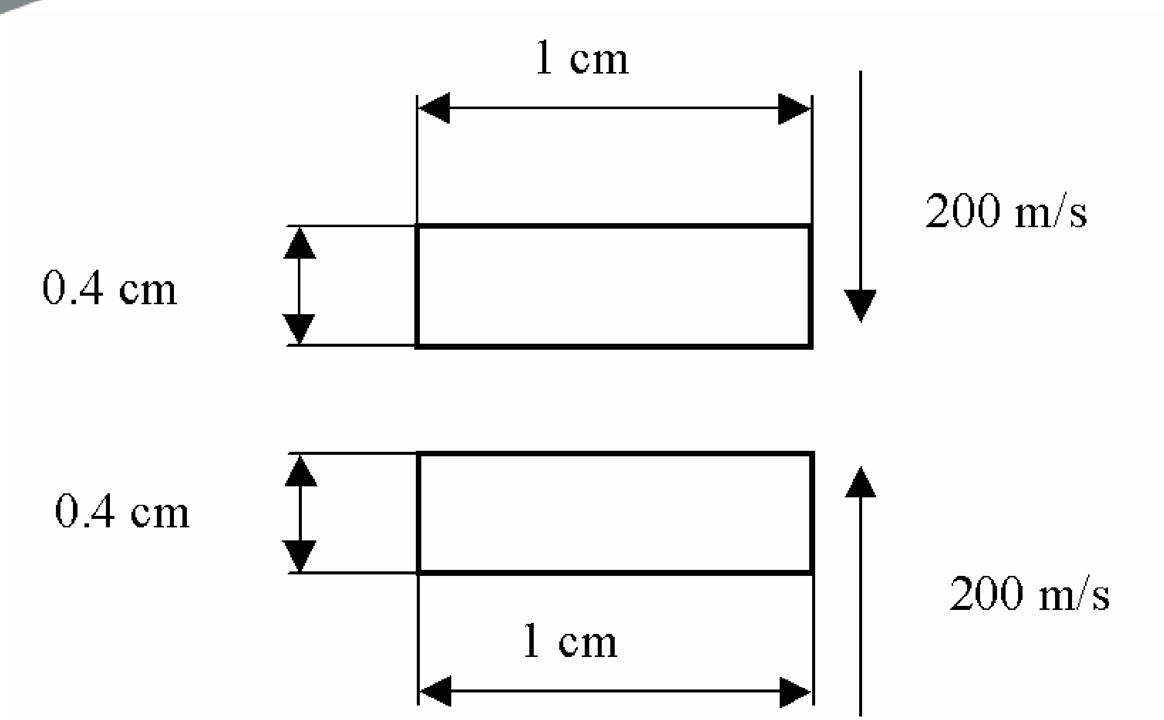
$$b_c(x_i) = \nabla\phi(x_i) = \sum_j^{NCONT} \frac{m_j}{\rho_j} Kn \frac{W(r_{ij})^{n-1}}{W(\Delta p_{avg})^n} \nabla_{x_i} W(x_i - x_j)$$

Normal Contact - SPH



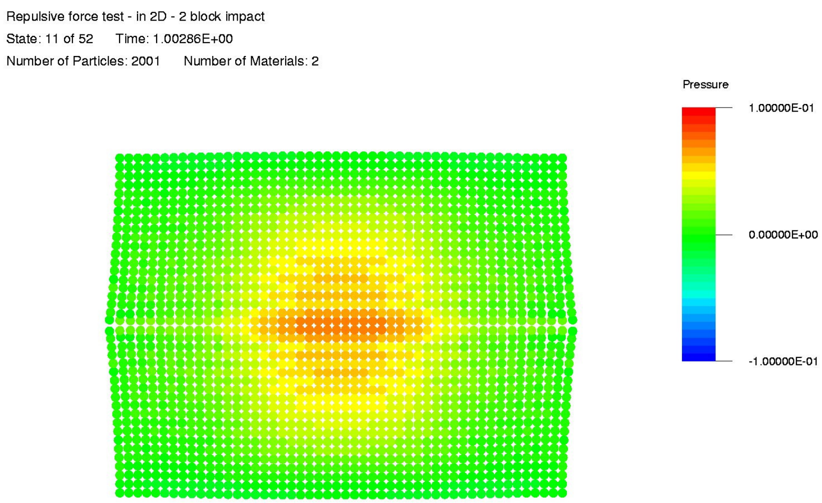
Example, Plate Impact (2D)

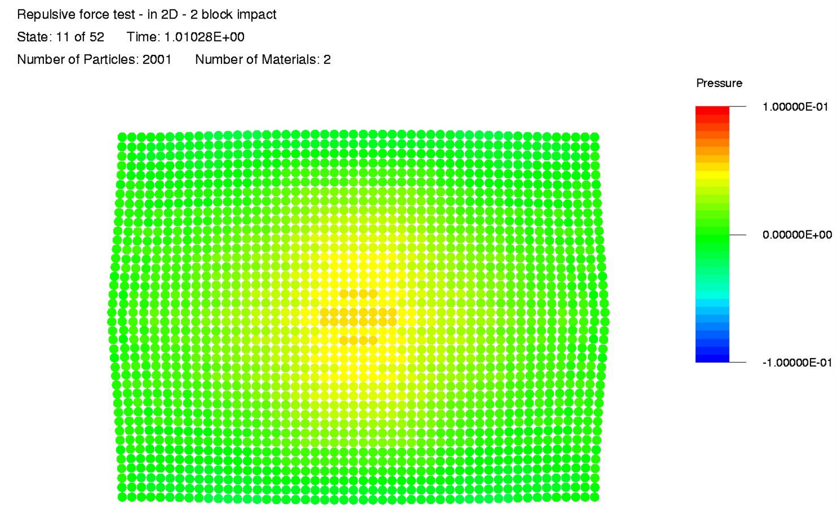
Cranfield UNIVERSITY

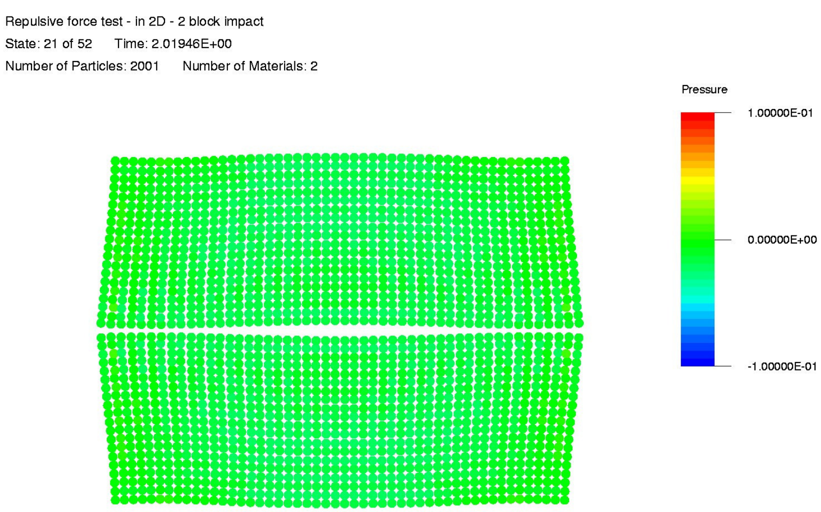


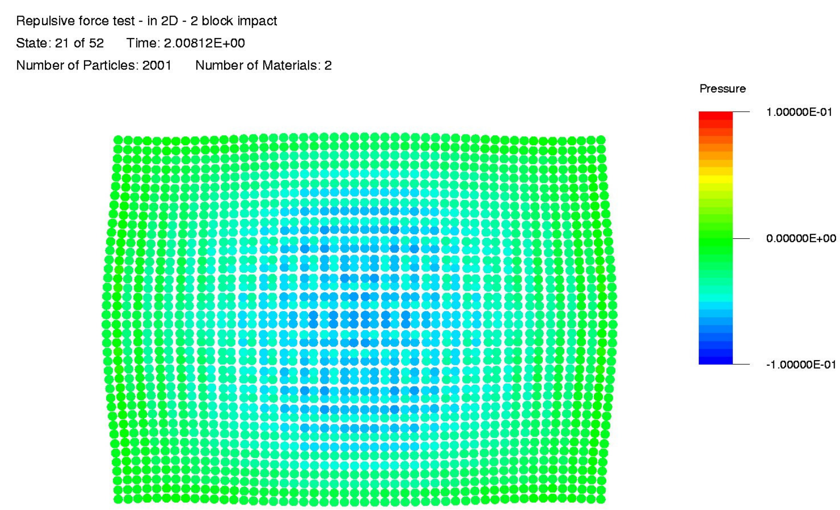
The symmetrical block impact
Each block was discretised with 50 by 20 particles
Steel blocs modelled with an elastic-plastic material model

Example, Plate Impact (2D)

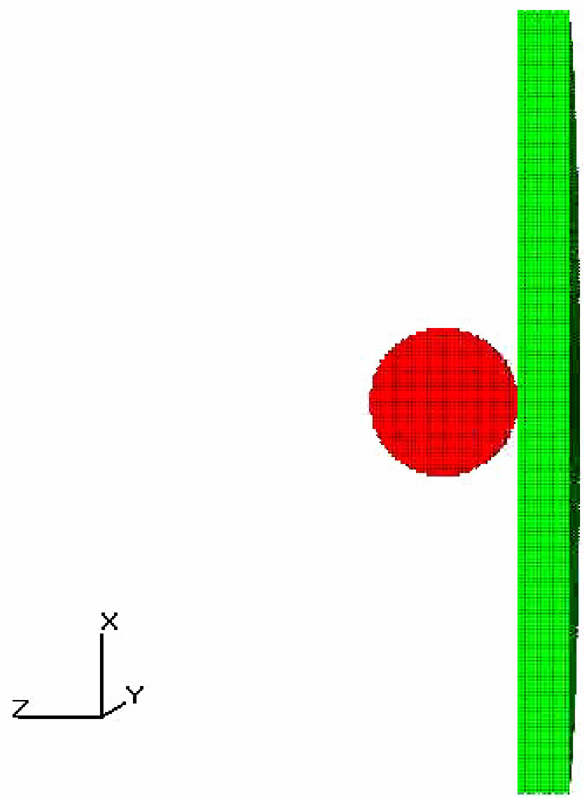








Example, Hypervelocity Impact (3D)



7km/s impact of aluminium projectile on aluminium bumper plate

Contact Algorithm Summary

Cranfield UNIVERSITY

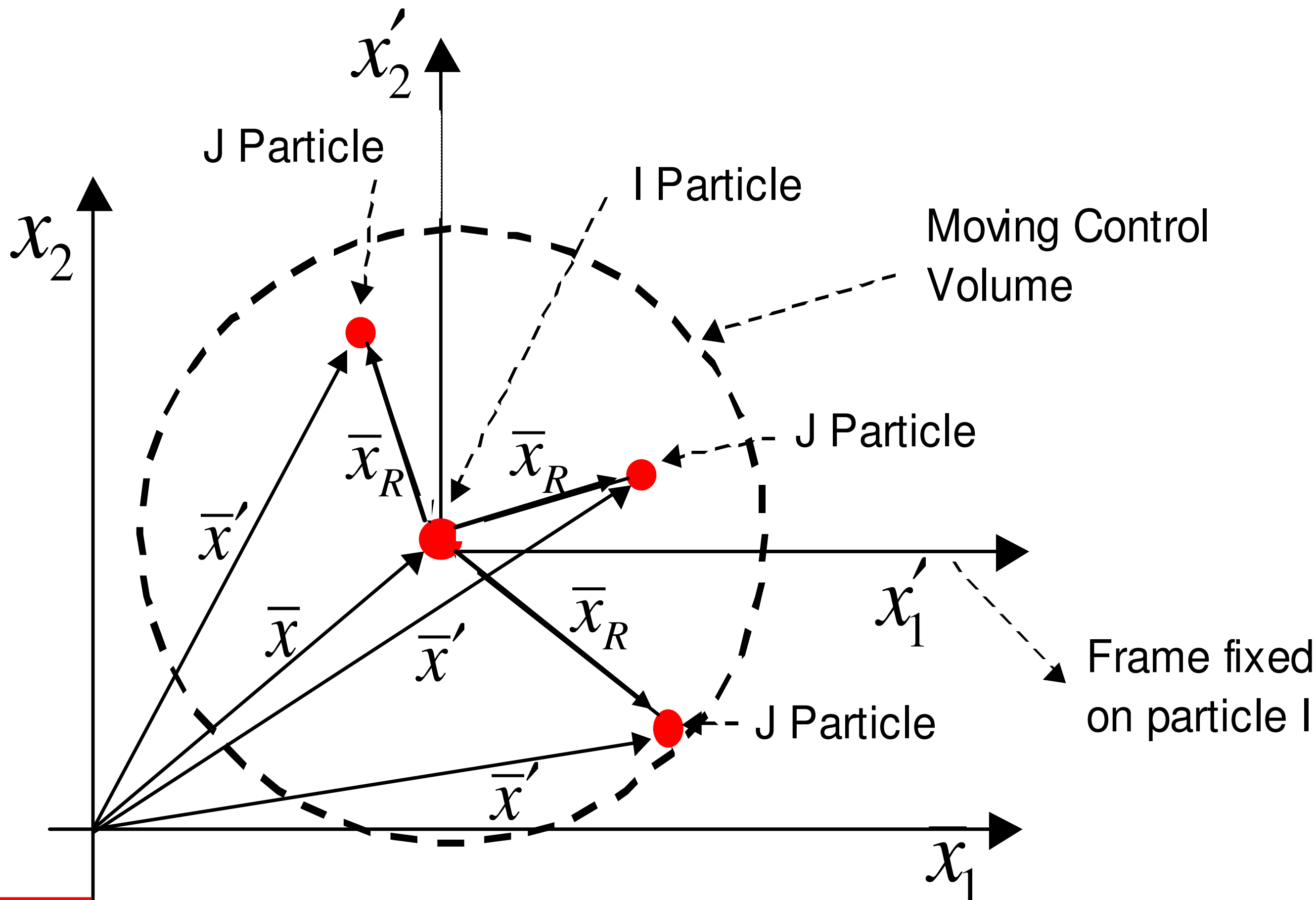
- Well suited for meshless methods
- Numerically effective, approximately additional 10% CPU time in 3D simulations
- Easy to add friction (ongoing work)

Alternative Form of the SPH Equations

R. Vignjevic, J. Campbell , J. Jaric, S. Powell; Derivation of SPH equations in a moving referential coordinate system, Comput. Methods Appl. Mech. Engrg. 198 (2009), pp. 2403-2411, (Received 8 September 2008)

Moving Referential Configuration

Cranfield UNIVERSITY



$$\underbrace{\frac{\tilde{D}f}{Dt}}_{referential} = \frac{Df}{Dt} + \vec{\mathbf{v}}_R \nabla \cdot f$$

$$\vec{\mathbf{v}}_R = \vec{\mathbf{v}}_J - \vec{\mathbf{v}}_I - \dot{\vec{h}}$$

Moving Referential Configuration

Cranfield UNIVERSITY

Conservation of Mass

$$\frac{\tilde{D}\rho}{Dt} + \rho\nabla\cdot\vec{\mathbf{v}} + \vec{\mathbf{v}}_R\cdot\nabla\rho = \frac{\tilde{D}\rho}{Dt} + \rho\nabla\cdot\vec{\mathbf{v}} + (\vec{\mathbf{v}} - \vec{\mathbf{v}}_{CV})\cdot\nabla\rho =$$

$$\frac{\tilde{D}\rho}{Dt} + \nabla\cdot(\rho\mathbf{v}) - \mathbf{v}_{CV}\cdot\nabla\rho = 0$$

$$\int_\Omega \frac{\tilde{D}\rho}{Dt} W_{IJ}\, d\Omega + \int_\Omega \nabla(\rho\vec{\mathbf{v}}) W_{IJ}\, d\Omega + \int_\Omega \vec{\mathbf{v}}_{CV}\nabla\rho W_{IJ} = 0$$

Integrating by parts and neglecting the boundary terms

$$\langle\dot{\rho}\rangle = \int_\Omega \rho\mathbf{v}\cdot\nabla W(|\mathbf{x}' - \mathbf{x}|, h)\, d\Omega - \mathbf{v}_I \int_\Omega \rho\nabla W(|\mathbf{x}' - \mathbf{x}|, h)\, d\Omega$$

$$\langle\dot{\rho}\rangle = -\sum_J m_J(\mathbf{v}_J - \mathbf{v}_I)\cdot\nabla(W_{IJ})$$

Moving Referential Configuration

Cranfield UNIVERSITY

Conservation of Momentum

$$\frac{\partial(\rho\vec{\mathbf{v}})}{\partial t} + \nabla\left(\rho\vec{\mathbf{v}} \otimes \vec{\mathbf{v}}_R\right) = \rho\left[\frac{\tilde{D}\vec{\mathbf{v}}}{Dt} + \nabla\cdot\vec{\mathbf{v}}\,\vec{\mathbf{v}}_R\right] = \nabla\cdot\sigma$$

$$\int_\Omega \frac{\tilde{D}\vec{\mathbf{v}}}{Dt} W_{IJ}\, d\Omega + \int_\Omega \nabla\cdot\vec{\mathbf{v}}\,\vec{\mathbf{v}}_R W_{IJ}\, d\Omega = \frac{1}{\rho}\int_\Omega \nabla\cdot\sigma W_{IJ}\, d\Omega$$

Using the divergence theorem, dropping the boundary terms and linearizing the velocity in the second term:

$$\left\langle \dot{\vec{\mathbf{v}}} \right\rangle + \vec{\mathbf{v}}\int_\Omega \nabla\vec{\mathbf{v}}_R W_{IJ}\, d\Omega - \int_\Omega \vec{\mathbf{v}}\left(\vec{\mathbf{v}}_R\cdot\nabla W_{IJ}\right) d\Omega = \frac{1}{\rho}\int_\Omega \nabla\cdot\sigma W_{IJ}\, d\Omega$$

Moving Referential Configuration

Conservation of Momentum

$$\left\langle \dot{\vec{\mathbf{v}}} \right\rangle + \vec{\mathbf{v}} \int_{\Omega} \nabla \vec{\mathbf{v}}_R W_{IJ} d\Omega - \int_{\Omega} \vec{\mathbf{v}} \left(\vec{\mathbf{v}}_R \cdot \nabla W_{IJ} \right) d\Omega = \frac{1}{\rho} \int_{\Omega} \nabla \cdot \sigma W_{IJ} d\Omega$$

When discretised, the above equation becomes:

$$\left\langle \dot{\vec{\mathbf{v}}}_I \right\rangle - \sum_{J} \left(\vec{\mathbf{v}}_J - \vec{\mathbf{v}}_I \right) \frac{m_J}{\rho_J} \left(\vec{\mathbf{v}}_R \cdot \nabla W_{IJ} \right) = -\frac{1}{\rho_I} \sum_{J \in \Omega} \left(\sigma_I + \sigma_J \right) \frac{m_J}{\rho_J} \bar{\bar{\nabla}}_{x'} W_{IJ}$$

Comparison

Cranfield UNIVERSITY

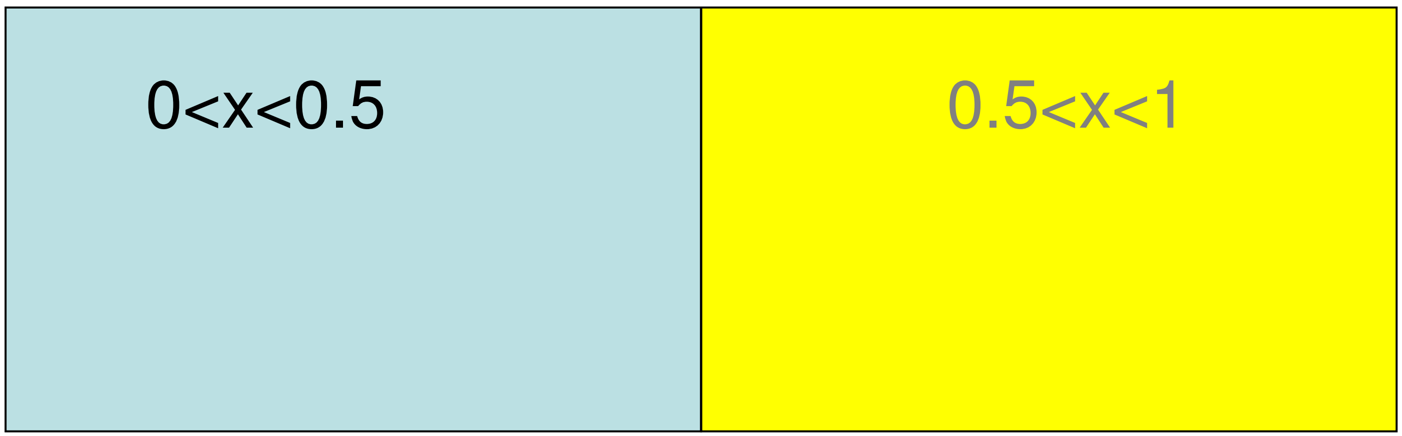
Continuity equation	
Conv. SPH [1, 2]	$\left\langle \dot{\rho}_I \right\rangle = \rho_I \sum_{J\in\Omega} \left(\vec{\mathbf{v}}_I - \vec{\mathbf{v}}_J \right) \nabla_x W_{IJ} \frac{m_J}{\rho_J}$
Moving C. S.	$\left\langle \dot{\rho} \right\rangle = \sum_J m_J \left(\vec{\mathbf{v}}_J - \vec{\mathbf{v}}_I \right) \nabla W_{IJ}$
Momentum equation	
Conv. SPH [1, 2]	$\left\langle \dot{\vec{\mathbf{v}}}_I \right\rangle = -\frac{1}{\rho_I} \sum_{J\in\Omega} \left(\sigma_I + \sigma_J \right) \frac{m_J}{\rho_J} \nabla_{x'} W_{IJ}$
Moving C. S.	$\left\langle \dot{\vec{\mathbf{v}}}_I \right\rangle - \sum_J \left(\vec{\mathbf{v}}_J - \vec{\mathbf{v}}_I \right) \frac{m_J}{\rho_J} \left(\vec{\mathbf{v}}_R \cdot \nabla W_{IJ} \right) = -\frac{1}{\rho_I} \sum_{J\in\Omega} \left(\sigma_I + \sigma_J \right) \frac{m_J}{\rho_J} \nabla_{x'} W_{IJ}$

1.- **Larry D. Libersky** at. al. *High Strain Lagrangian Hydrodynamics a 3-DSPH code for dynamic material response*,. Journal of Computational Physics, 1993

2.- **Morris, J.P.,** *An Overview of the Method of Smoothed Particle Hydrodynamics*, November 1995, Universitat Kaiserlautern, Arbeitsgruppe Technomathematik.

Numerical Example

Shock tube problem

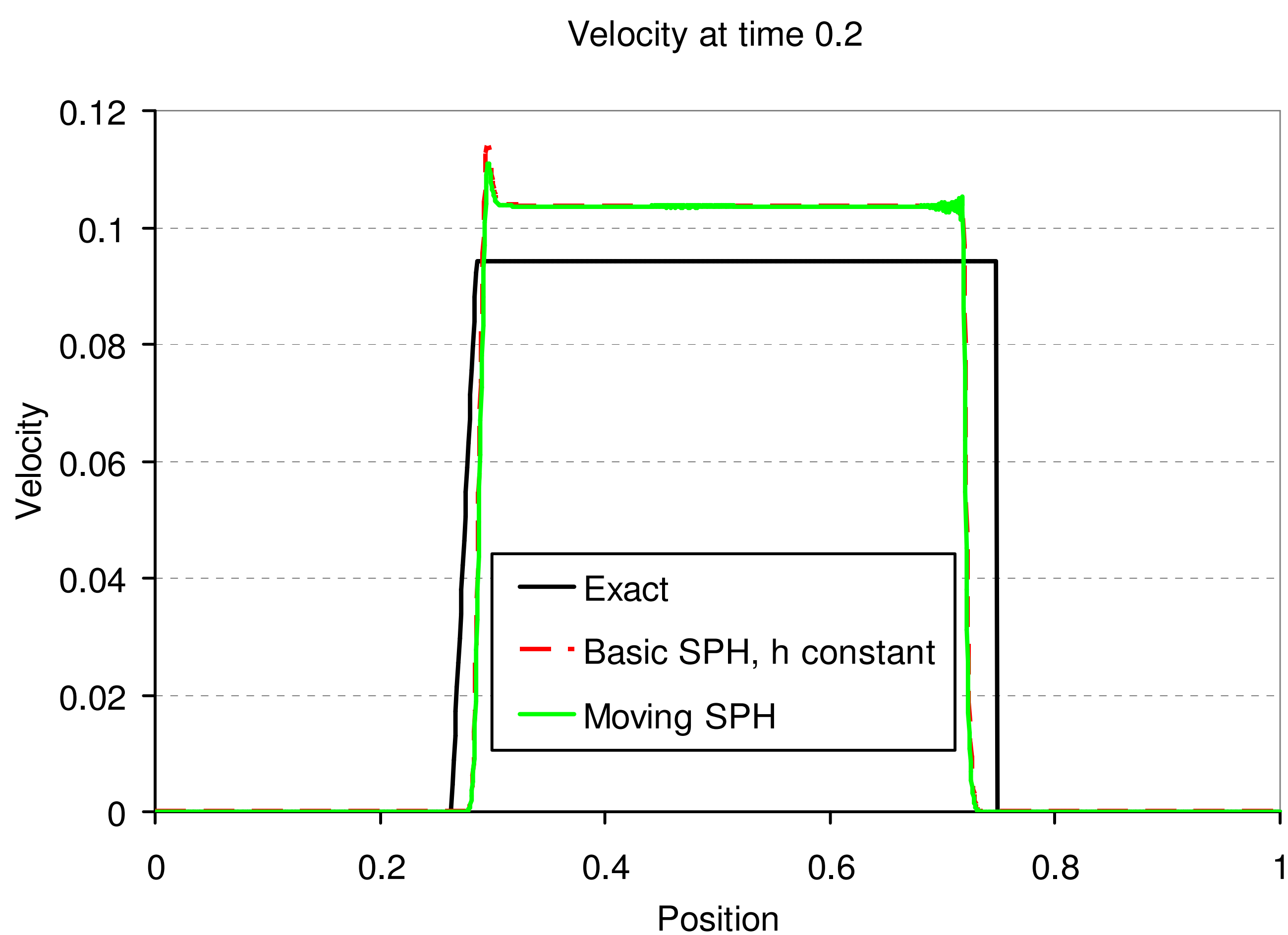


Initial conditions

	0<x<0.5 Left	0.5<x<1 Right
Density	1.0	0.8
Pressure	1.0	0.8
Velocity	0.0	0.0

Numerical Example

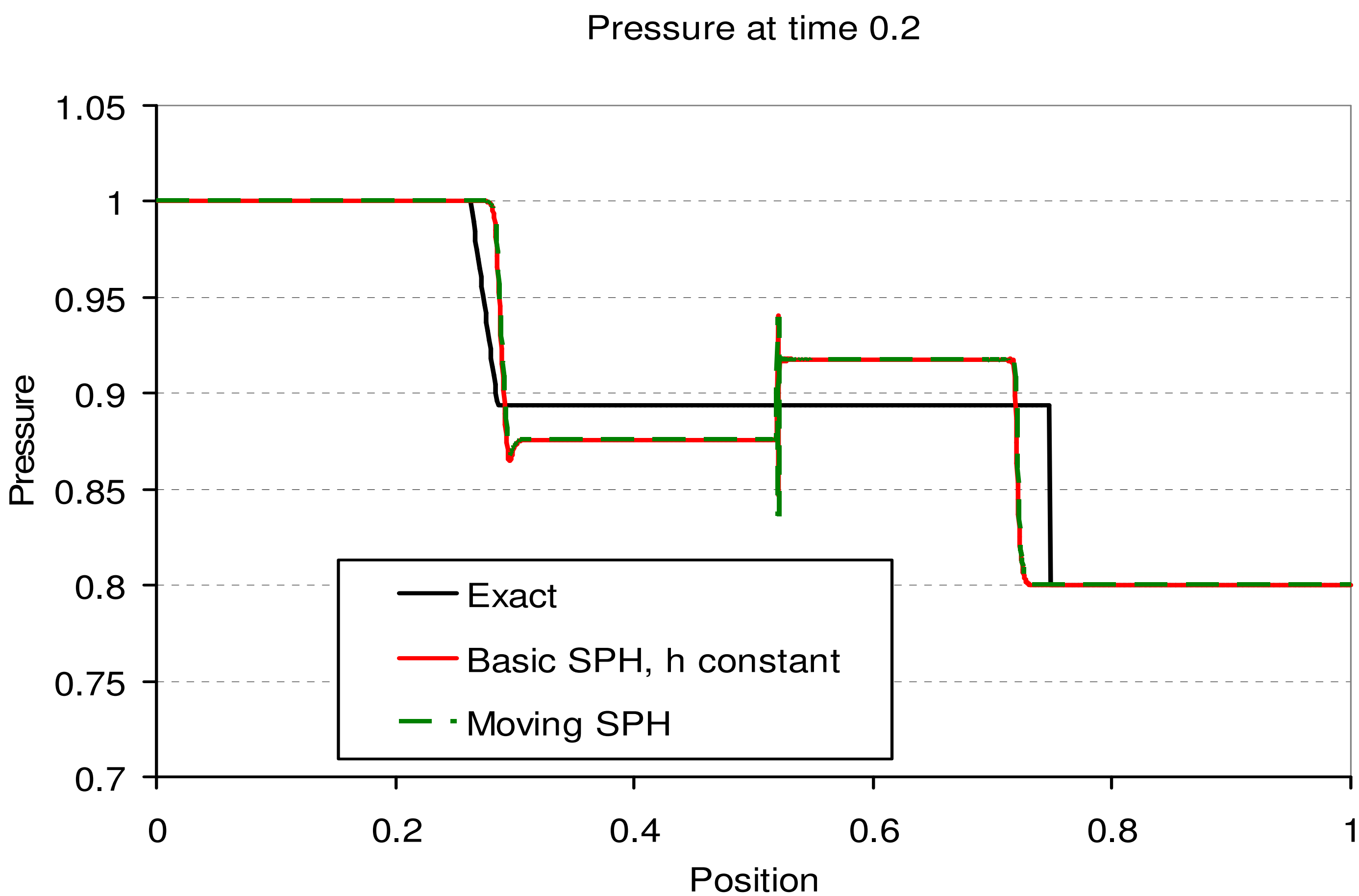
Shock tube problem



Numerical Example

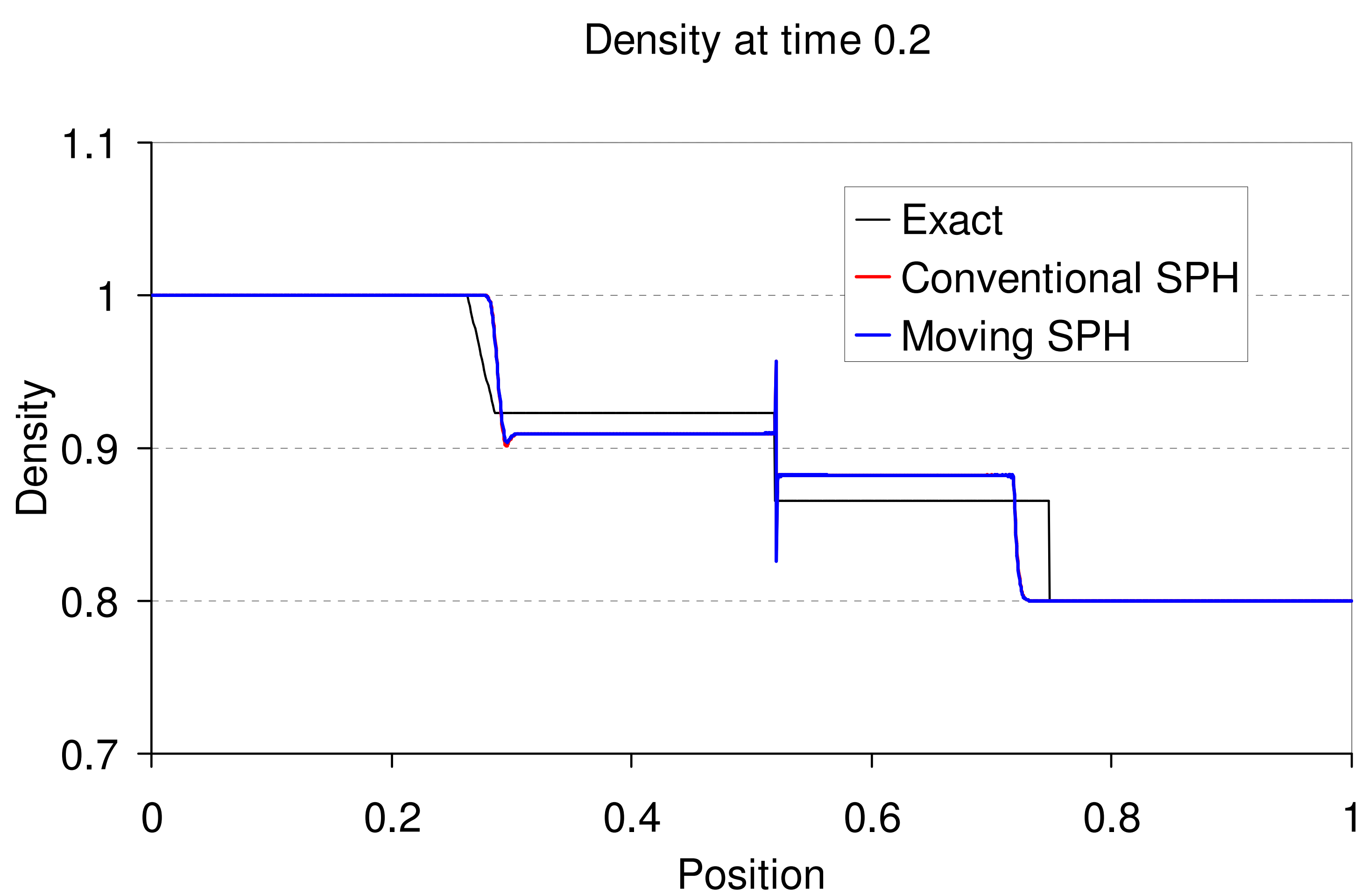
Cranfield UNIVERSITY

Shock tube problem



Numerical Example

Shock tube problem



Alternative Form of the SPH Equations Summary

Cranfield UNIVERSITY

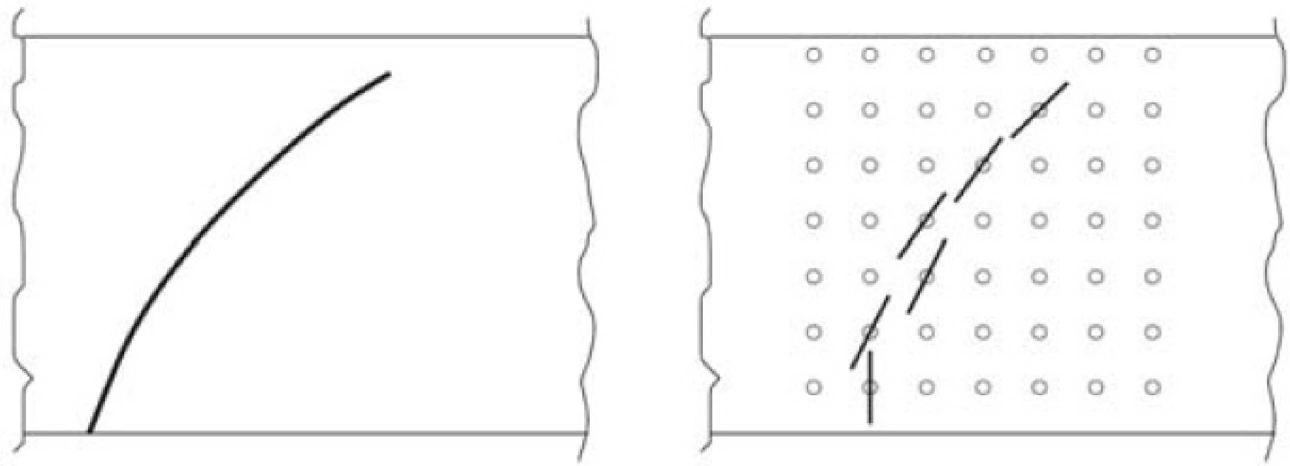
- Part of the ongoing work
- Framework for a more rigorous treatment of variable h interpolation
- In the shock tube problem performance similar to the conventional SPH with constant h

Damage Modelling

The fracture model capable of simulating:

- Initiation and growth of damage
- Crack formation and propagation in an arbitrary direction
- Crack branching and crack joining, leading to fragmentation

How to represent a Crack



‘Cracked/failed’ particle concept: A failed particle is split into two particles, this approach has been demonstrated by Rabczuk and Belytschko (2007)

Continuum Damage Mechanics

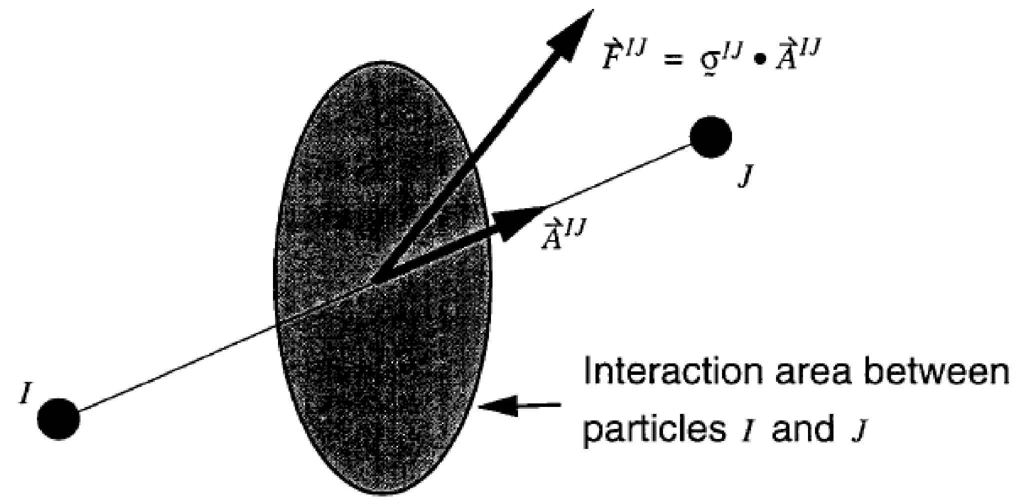
Cranfield UNIVERSITY

- Compatible with the concept of effective stress introduced by Kachanov in 1958
- The effective stress tensor is defined as:

$$\tilde{\sigma} = \frac{\sigma}{1 - D} \qquad D = \frac{S - S_0}{S_0}$$

- The concept of the interaction area relatively simple to apply within SPH

How to represent a Crack (Damage)



Inter-particle failure

Damage is evaluated for pairs of neighbour particles. Following fracture the particles cease being neighbours.

The concept of particle-particle interaction area (Swegle, 2000),

- damage affects interaction effective area
- at failure the effective area is at a critical value

Swegle Interaction Area

The force acting on a surface due to a stress is given by

$$\mathbf{F} = \boldsymbol{\sigma} \cdot \mathbf{A}$$

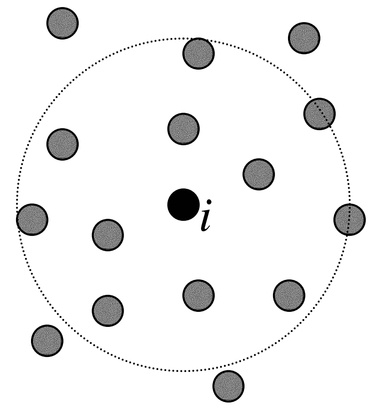
The SPH momentum equation could be rewritten in term of an interaction area:

$$a_i = \frac{dv_i}{dt} = \sum_j m_j \left[\frac{\sigma_i}{\rho_i^2} + \frac{\sigma_j}{\rho_j^2} \right] \nabla_i W_{ij}$$

$$m_i a_i = \sum_j vol_i vol_j \nabla_i W_{ij} \left[(\sigma_i) \frac{\rho_j}{\rho_i} + (\sigma_j) \frac{\rho_i}{\rho_j} \right] = F_i$$

$$= \sum_j \boxed{A_{ij}} \left[(\sigma_i) \frac{\rho_j}{\rho_i} + (\sigma_j) \frac{\rho_i}{\rho_j} \right]$$

Inter-particle failure



Cranfield UNIVERSITY

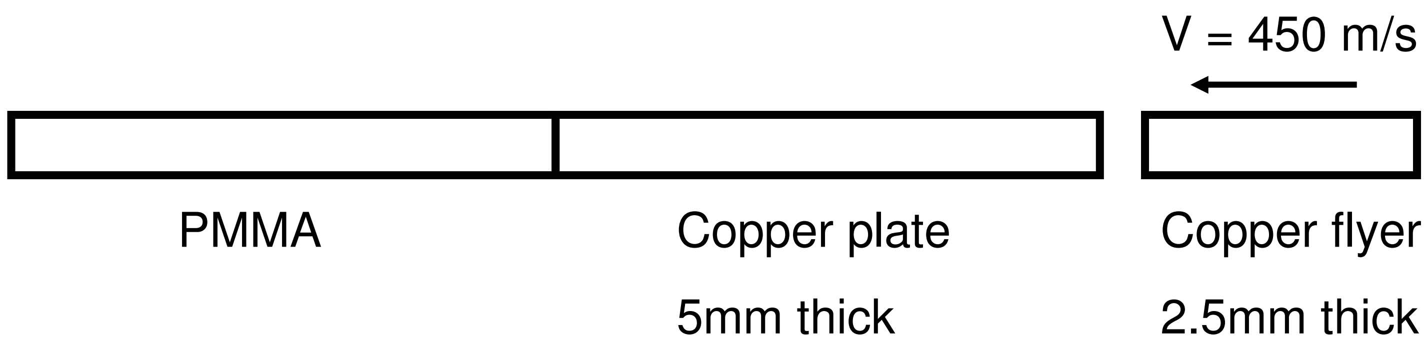
- Damage is evolved as an inter-particle value, D_{ij} ,which reduces the inter-particle interaction area:

$$F_i = \sum_j \left[(\sigma_i)\frac{\rho_j}{\rho_i} + (\sigma_j)\frac{\rho_i}{\rho_j} \right] A_{ij}\left(1 - D_{ij}\right)$$

- When damage reaches a critical value the material is assumed to have failed and the particles cease to be neighbours.

Spall demonstration problem

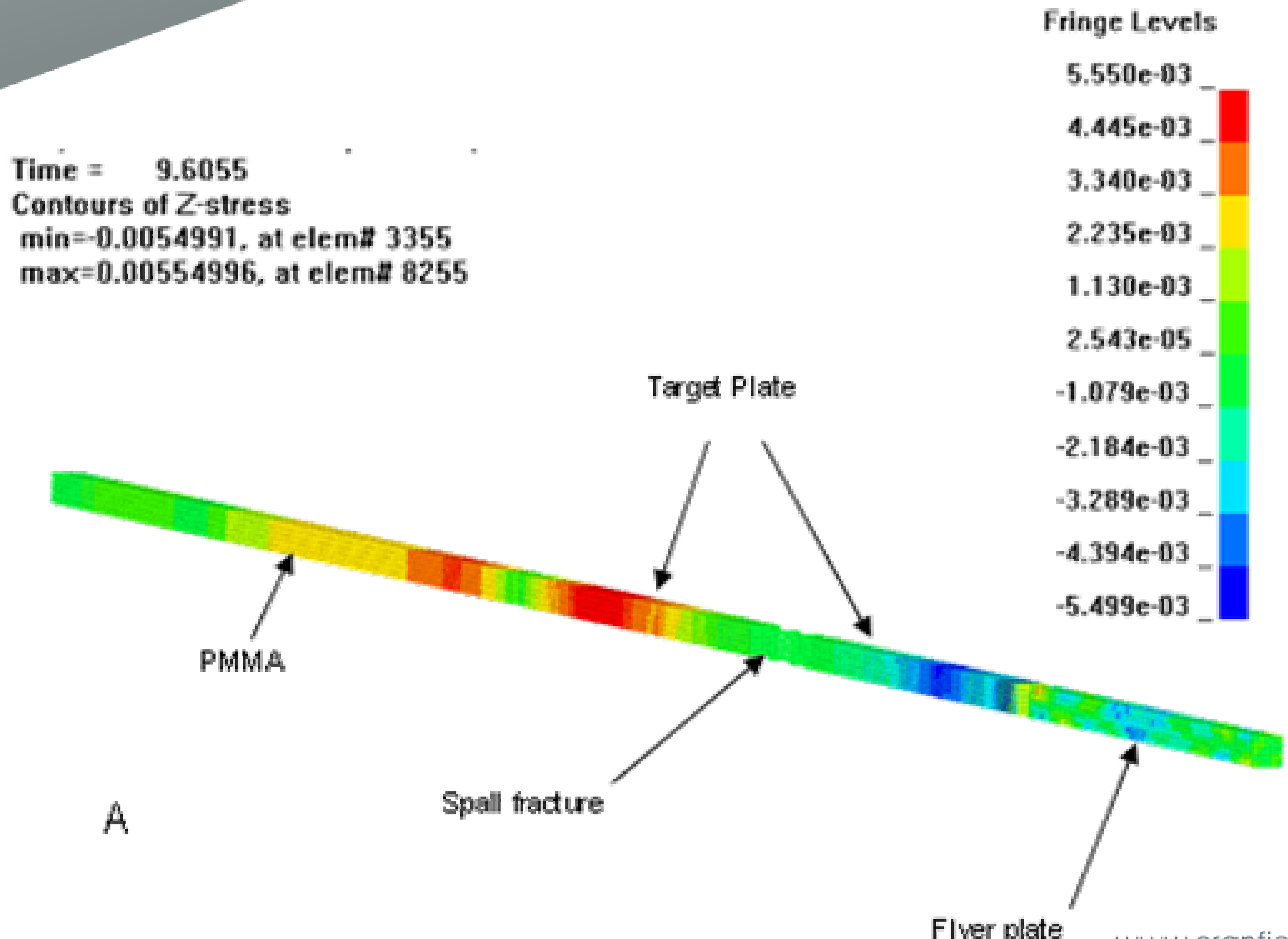
- Normal stress between pairs of particles is compared to a spall criterion. Damage initiated once this criterion is exceeded.



- 1D strain state
- Spall plane opens in middle of target plate

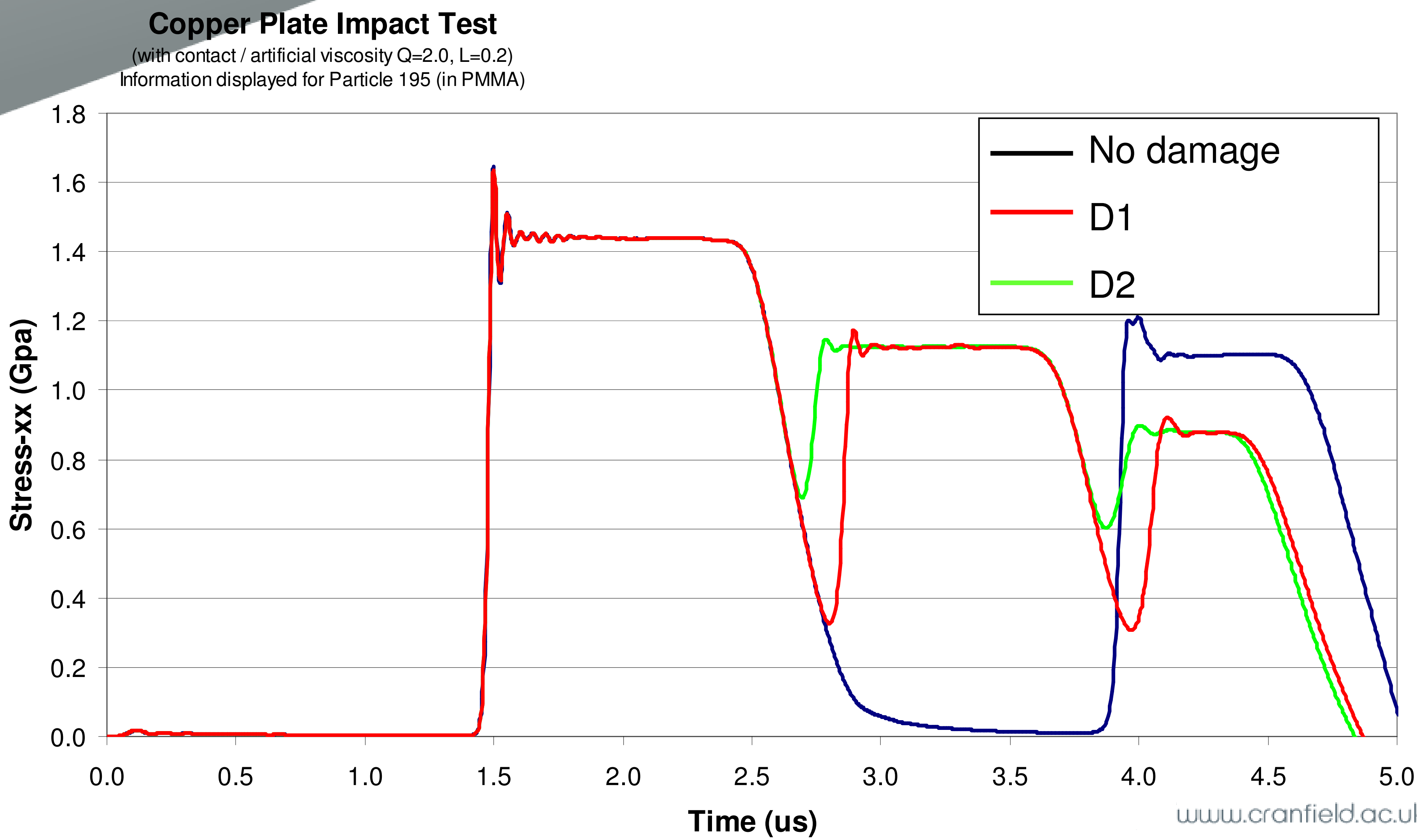
Representing fracture within a meshless method

Cranfield UNIVERSITY



Numerical Results

Cranfield UNIVERSITY

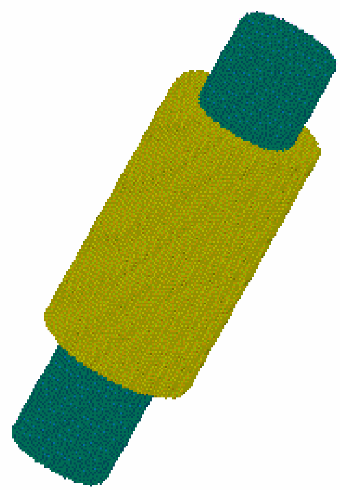


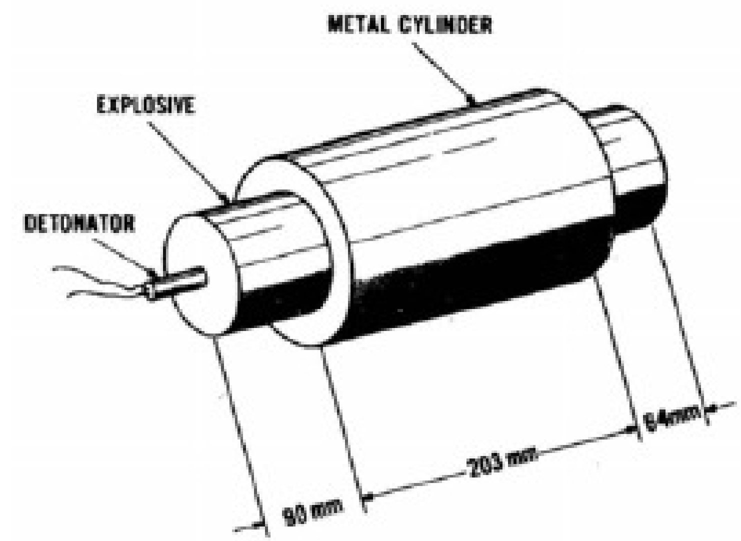
The Mock-Holt problem

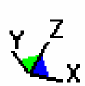
Cranfield UNIVERSITY

Coarse Mock-Holt Simulation - Standard
Time = 0

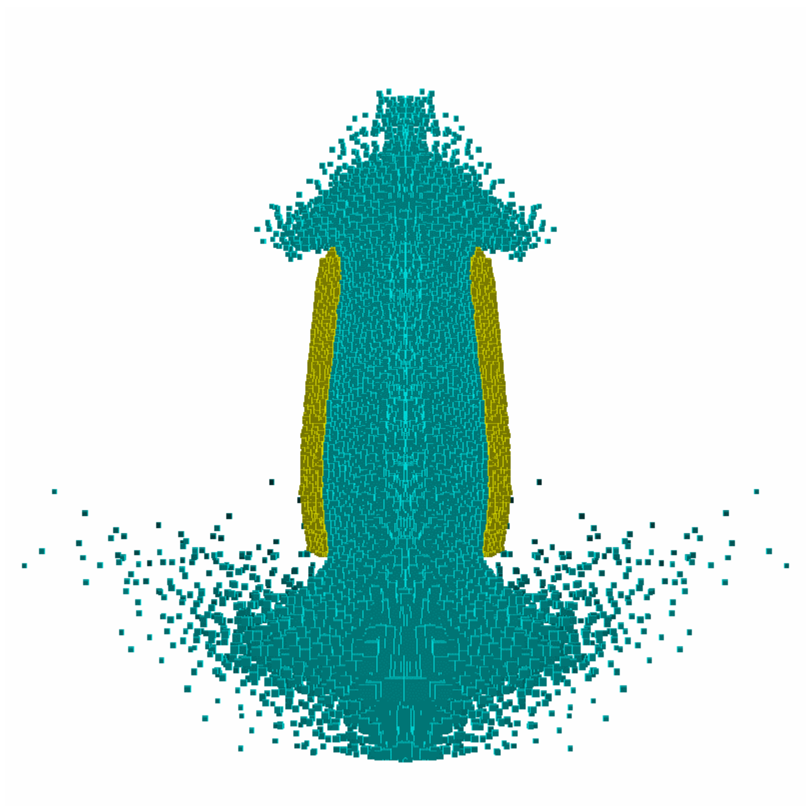
Mock and Holt (1983), experimental results from a number of tests on explosive driven Fragmentation of metallic cylinders.

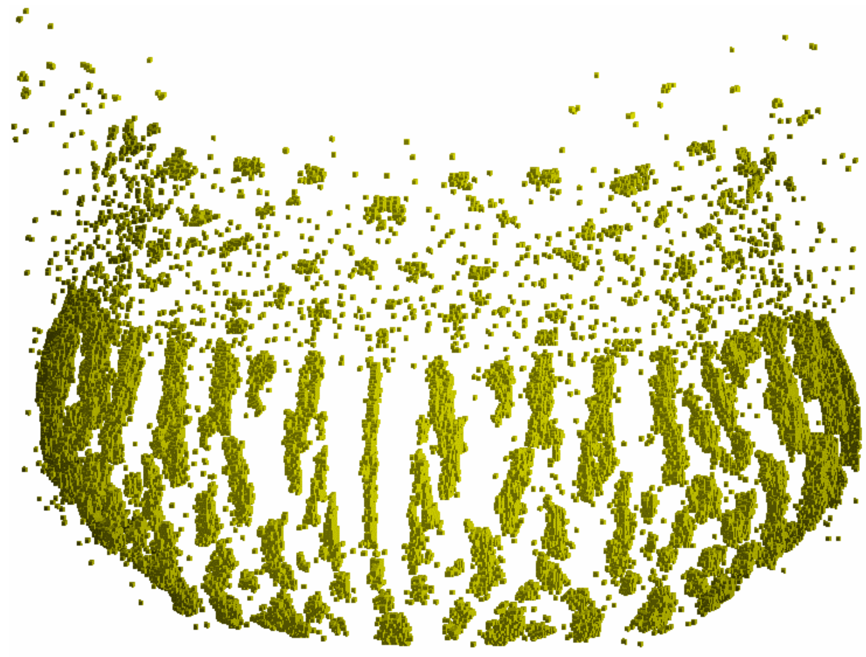






The Mock-Holt problem





Damage Modelling Summary

Cranfield UNIVERSITY

- Well suited for meshless methods
- Numerically effective
- Can deal with crack closure
- Ongoing work

Overall Summary

Cranfield UNIVERSITY

- All developments discussed were implemented into our 3D SPH code
- The SPH code coupled with DYNA3D
- The codes are routinely used to analyse real world problems
- Further development of the SPH method is of high priority to us (stabilisation of the Eulerian SPH, development of SPH structural elements, modelling damage,…)

Cranfield UNIVERSITY

The End