Cracking and crushing: Modeling the collisional history of small bodies in the solar system

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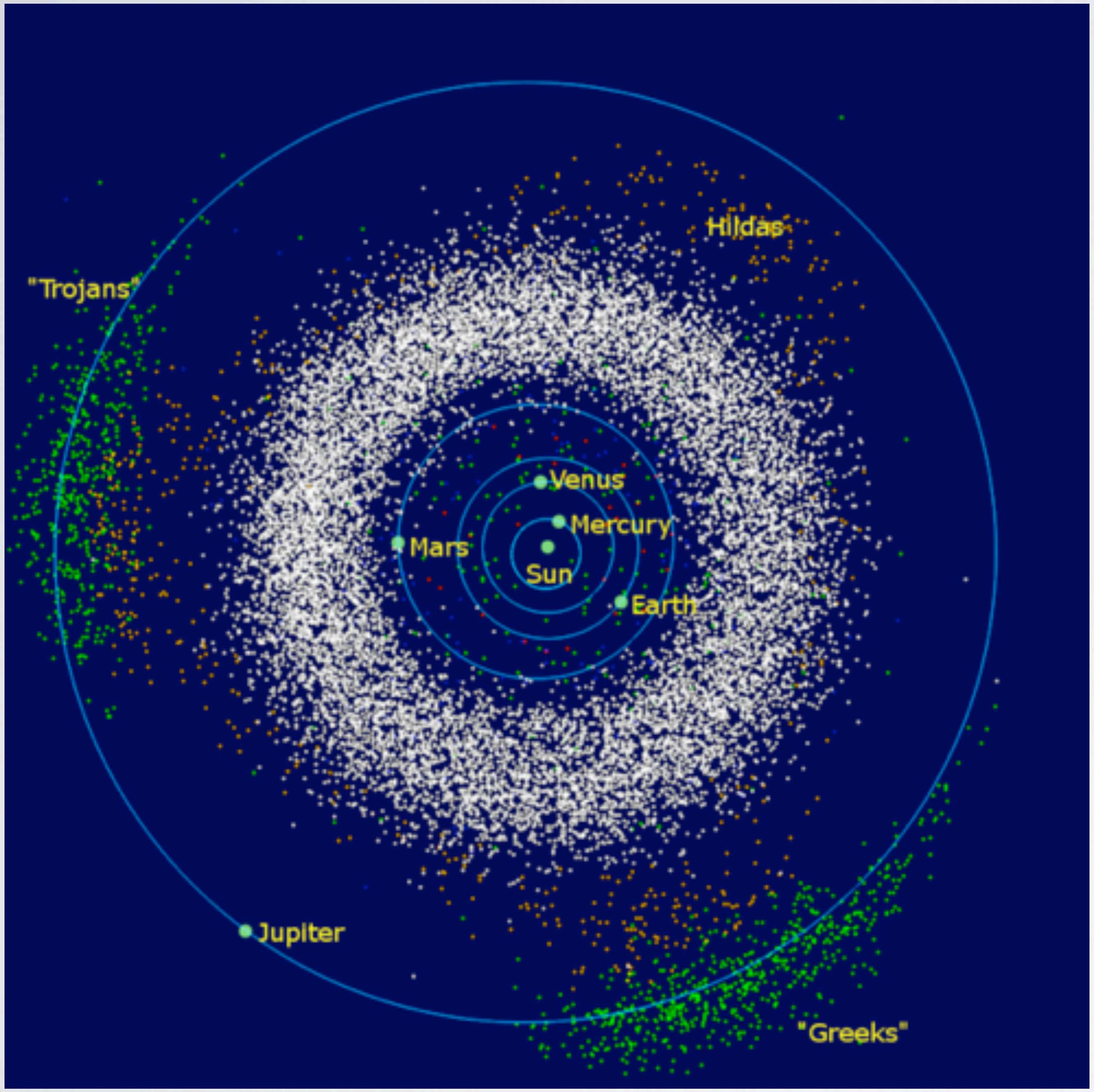
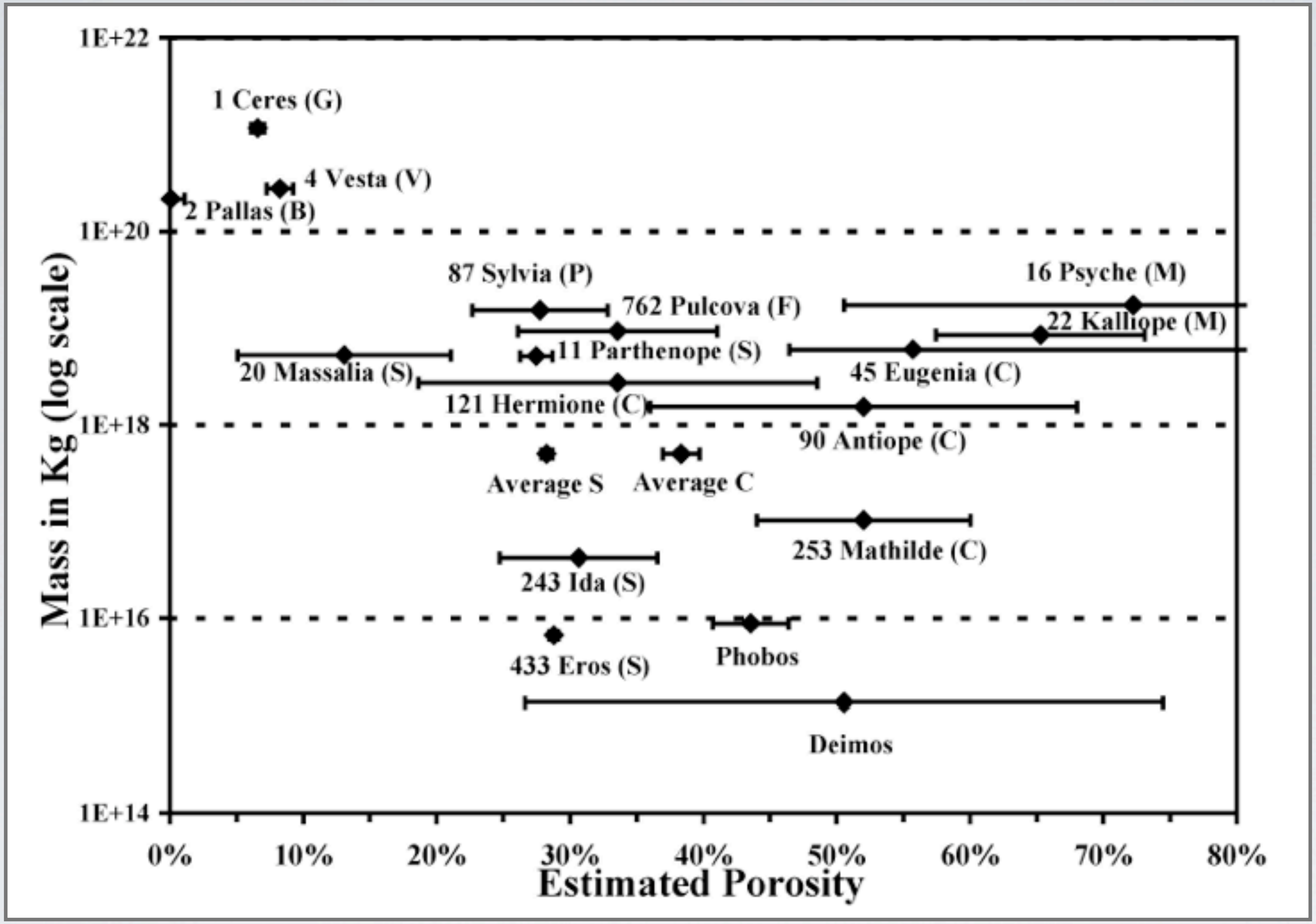
Small bodies: The asteroid population



Fact sheet:

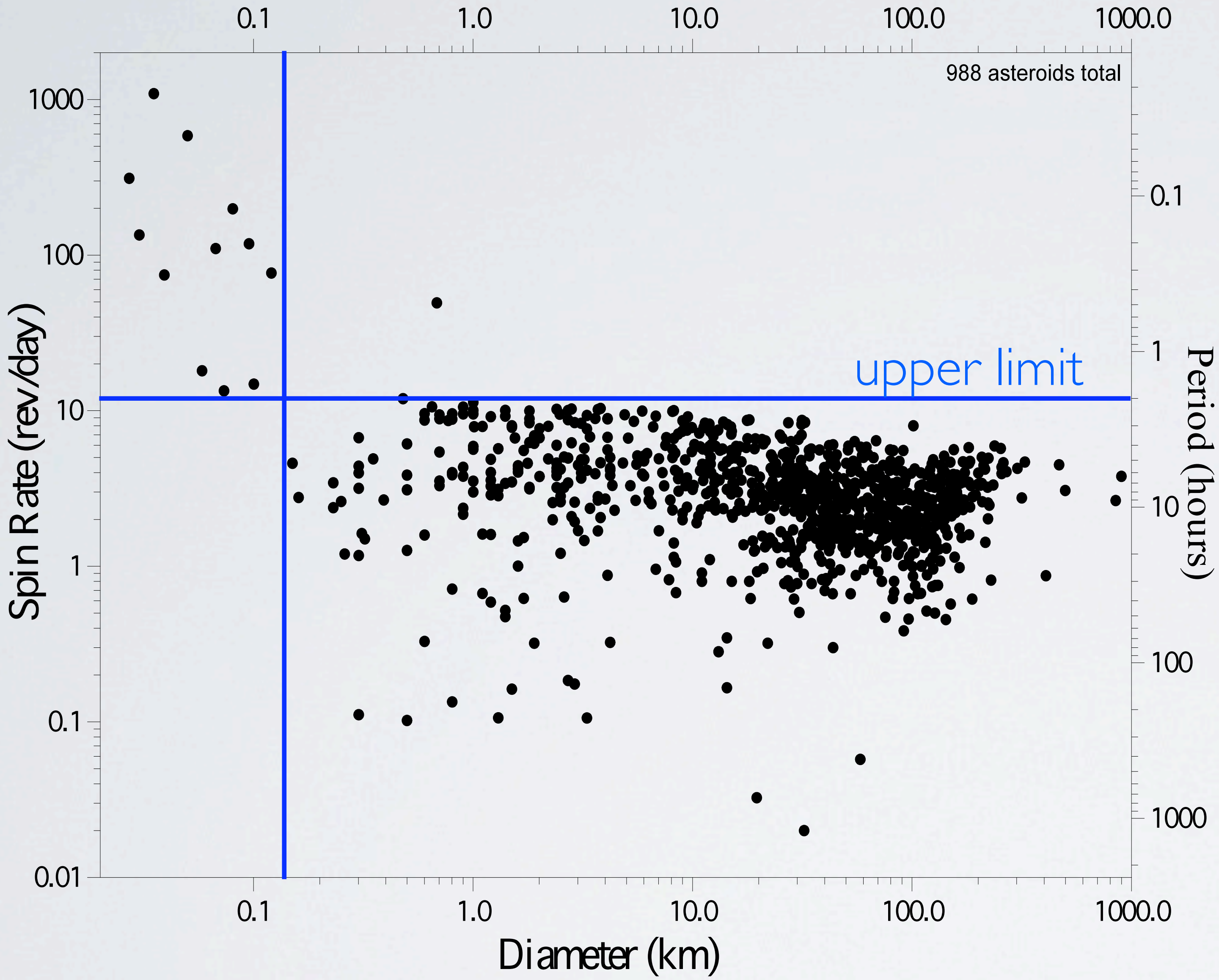
- *estimated number: 1.1-1.9 10^6 with diameter >1 km*
- *estimated total mass: 4% of a lunar mass*
- *orbits located mainly between Mars and Jupiter*
- *largest object: Ceres D=950 km*

Mean density of asteroids

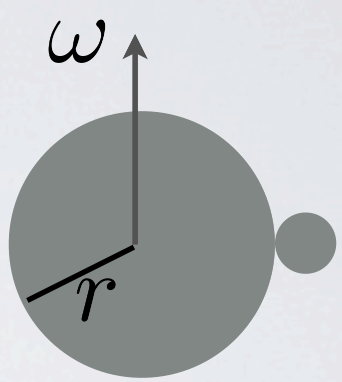


there is considerable evidence that asteroids are porous bodies...

Rotation rate of asteroids



rubble pile speed limit:



gravity = centrifugal force

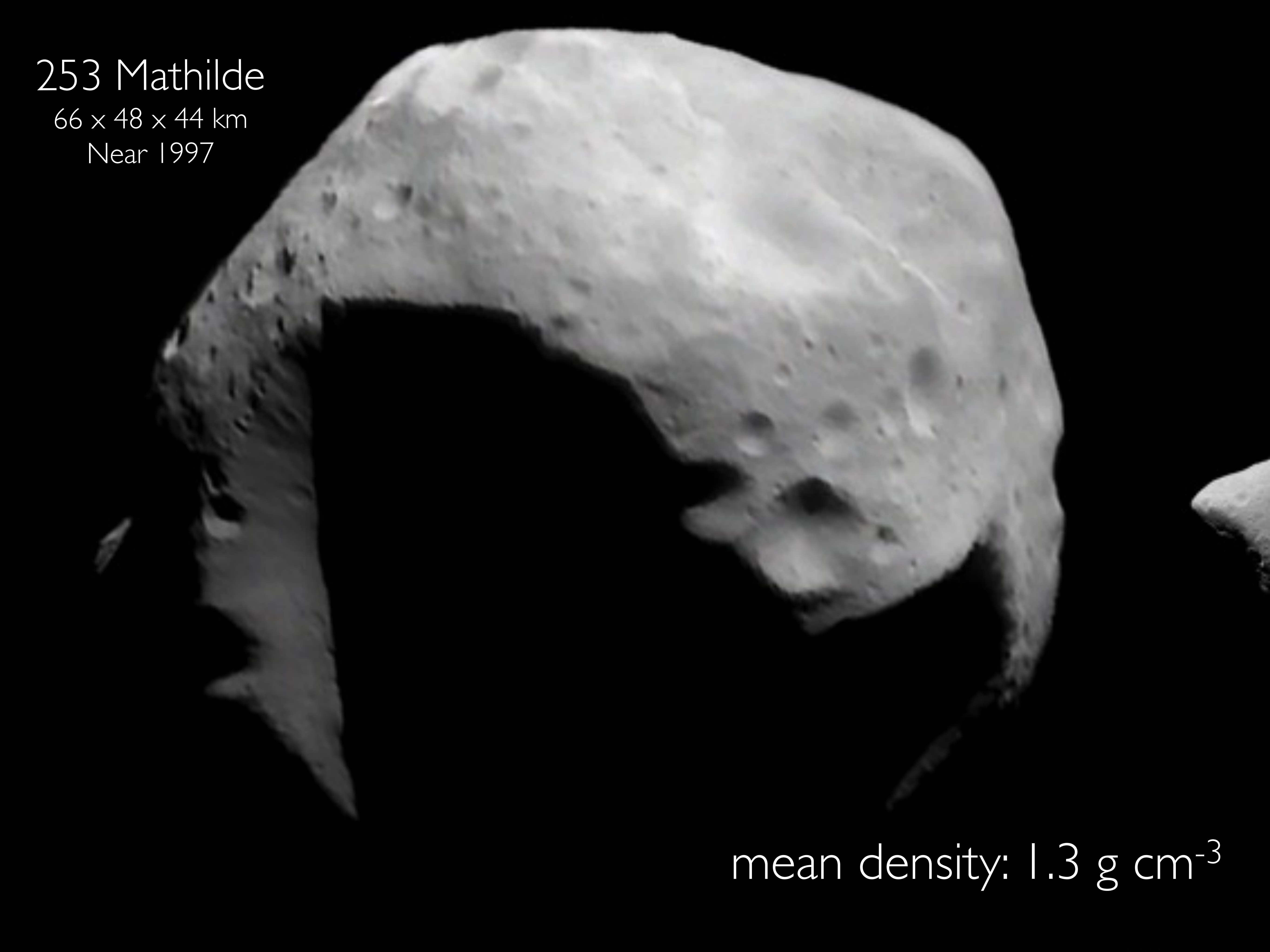
$$\omega = \sqrt{\frac{Gm}{r^3}} = \omega_0 \sqrt{\rho}$$

→limit independent of size...

there is considerable evidence that large asteroids are ruble piles...

Meet the small and famous...


Meet the small and famous...
433 Eros
33 x 13 km
Near 2000
5535 Annefrank
7 x 5 x 3 km
Stardust 2002
2867 Steins
6 x 5 x 5 km
Rosetta 2008
951 Gaspra
18 x 11 x 9 km
Galileo 1993
253 Mathilde
66 x 48 x 44 km
Near 1997
25143 Itokawa
0.5 x 0.3 x 0.2 km
Hayabusa 2005
9969 Braille
2 x 1 x 1 km
Deep Space 1 1999
Dactyl
1.6 x 1.2 km
Galileo 1993
1P/ Halley
16 x 8 x 8 km
Vega 2 1986
9P/ Tempel 1
8 x 5 km
Deep impact 2005
243 Ida
59 x 25 x 19 km
Galileo 1993
19P/ Borelly
8 x 4 km
Deep Space 1 2001
81P/ Wild 2
6 x 4 x 3 km
Stardust 2004


253 Mathilde
66 x 48 x 44 km
Near 1997
mean density: 1.3 g cm^{-3}

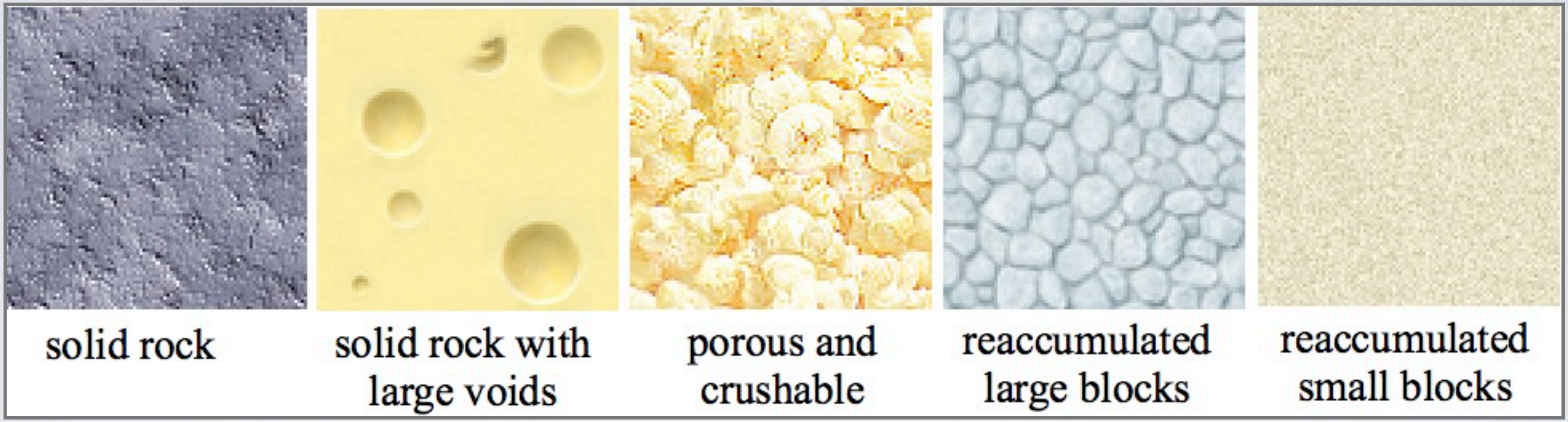
2867 Steins
6 x 5 x 5 km
Rosetta 2008

A diamond in the sky....

Modeling solid materials

(for planetary science applications)

What is out there:



the material characteristics will determine the response to impacts and hence will determine the evolution of the bodies

Fracture

- explicit flaw distribution: Weibull distribution

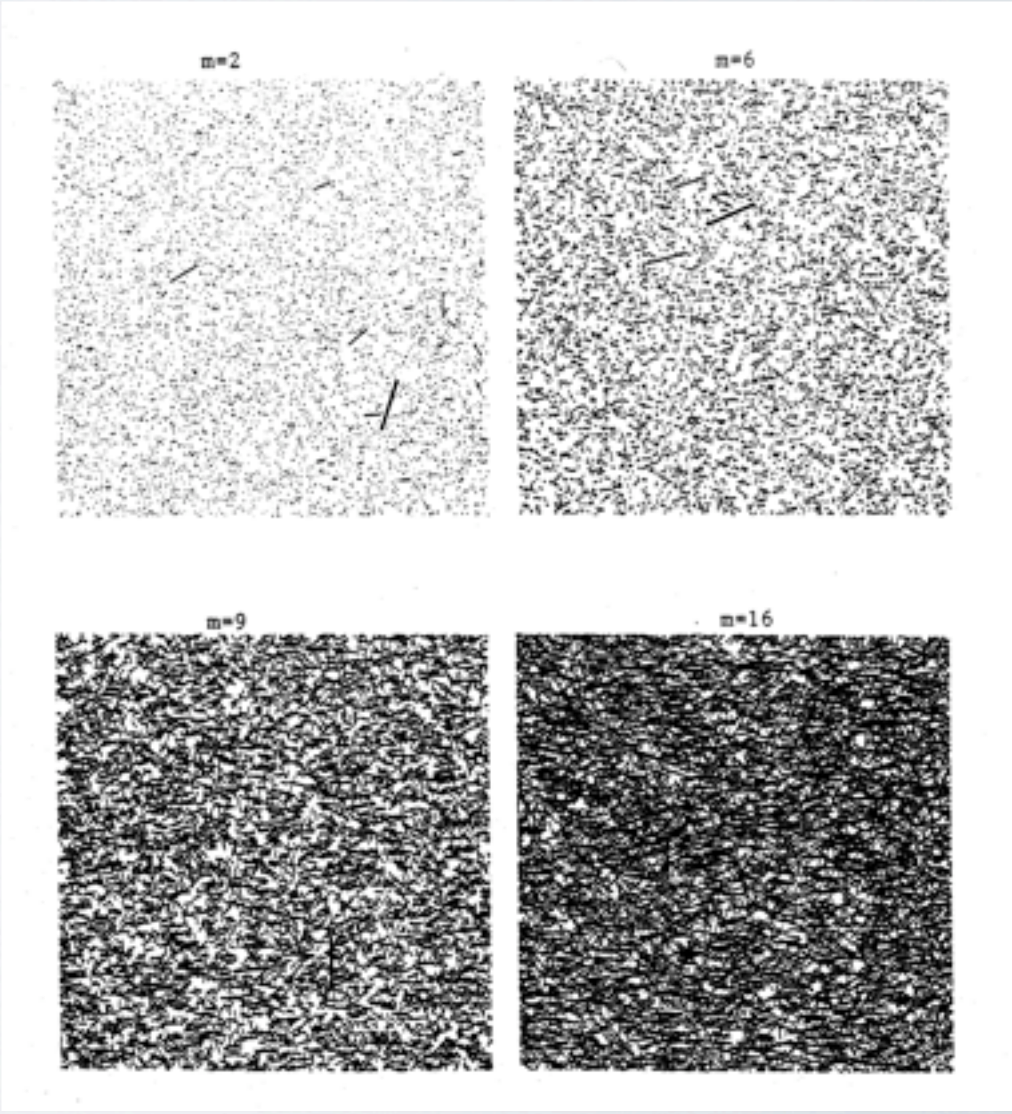
$n(\epsilon) = k\epsilon^m$ with $n(\epsilon)$ the number density of active flaws at strain ϵ

k, m material parameters

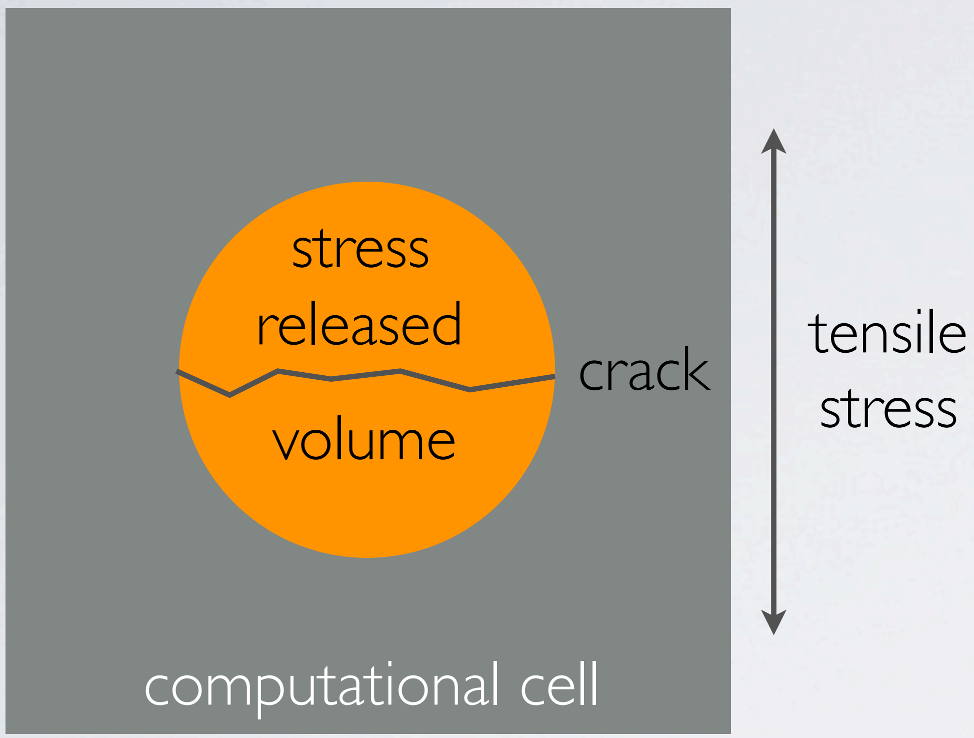
- flaw activation threshold

$$\epsilon_i = \left(\frac{i}{kV}\right)^{1/m} \quad i = 1, 2, \ldots, N$$

activation thresholds are distributed randomly over all particles



Damage: feed-back on the dynamics



Damage growth:

$$D(t) = \frac{V(t)}{V_{cell}} = \frac{\frac{4}{3}\pi c_g (t - t_0)}{V_{cell}}$$

with: t_0 the activation time

c_g the crack growth speed

$$0 \leq D \leq 1$$

intact → full tension and shear

entirely damaged → no tension or shear

Damage is the result of the entire stress history of a solid
→the Lagrangian nature of SPH is essential

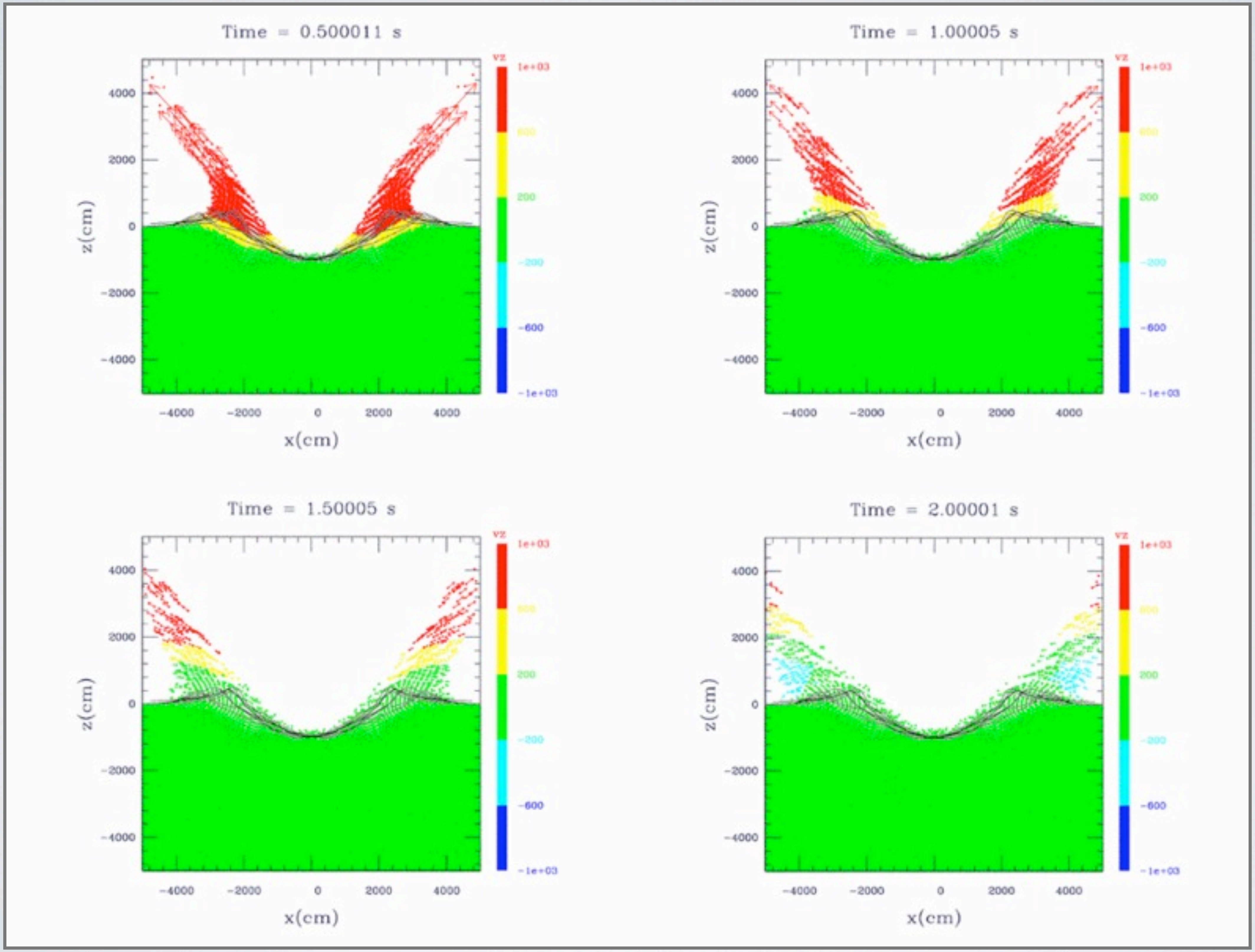
Comparison with experiments

→ sailor hat experiment (1965):
$4.54\ 10^8$ g of TNT detonated on Kahoolawe Island (Hawaii): $4.2\ 10^{10}$ ergs

crater diameter: 88.4 m

Sailor hat simulations

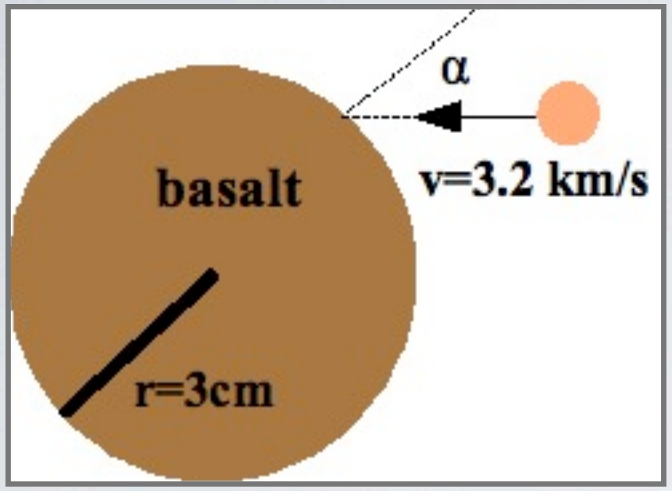
SPH simulations using 2.5×10^6 particles



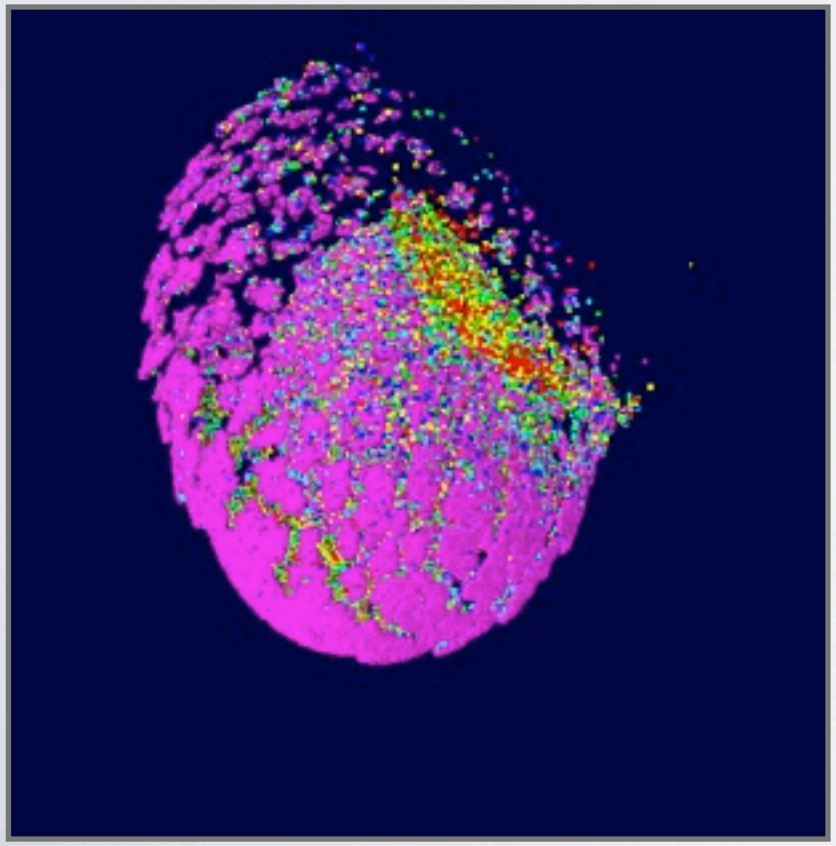
Comparison with impact experiments

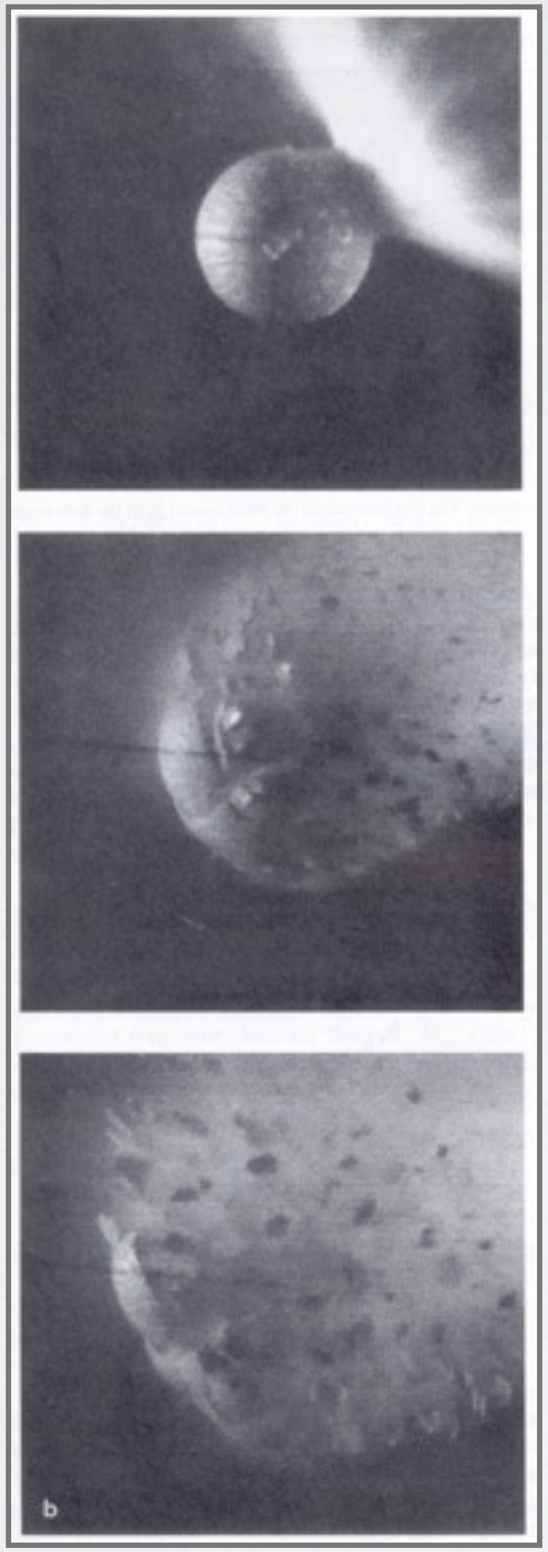
→ SPH simulations using 3×10^6 particles

Nakamura & Fujiwara 93

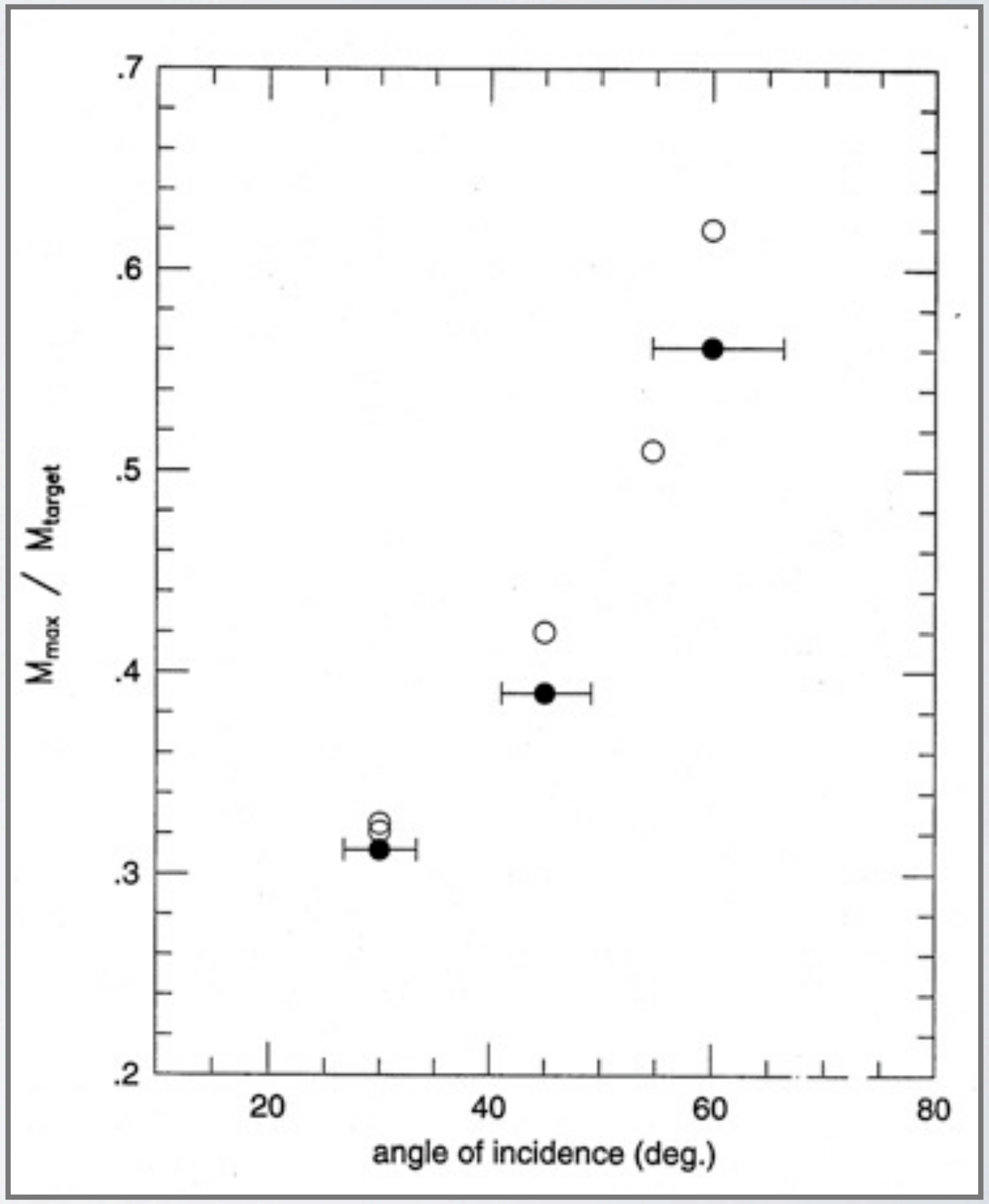


dust removed



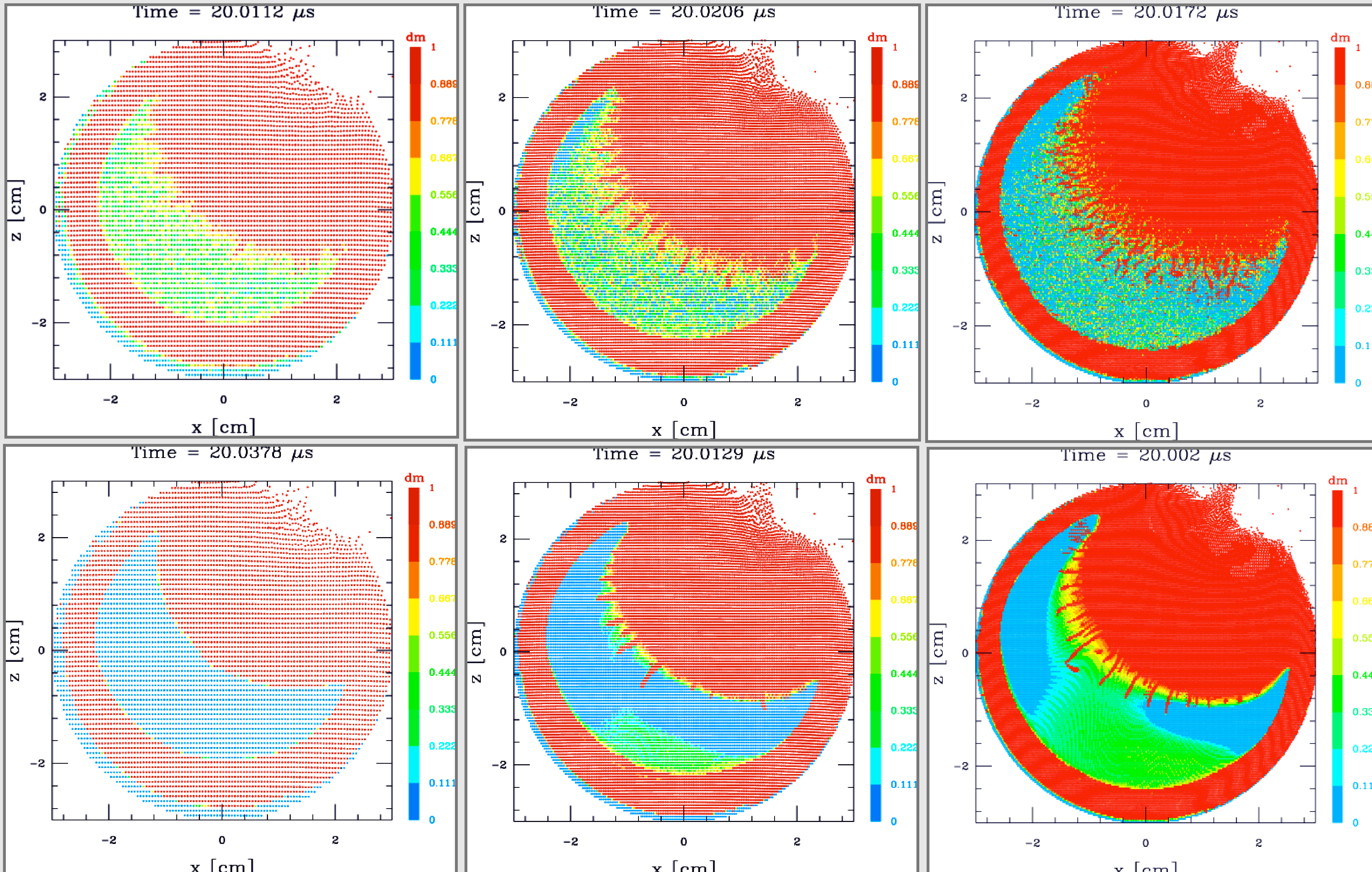


largest fragment as a function of impact angle



Convergence

	N=140’000 h=0.104 cm	N=700’000 h=0.061 cm	N=3’500’000 h=0.036 cm
m=9.5			
m=∞			



Porosity

Definitions:

- porosity:

$$\phi = \frac{V - V_S}{V} = \frac{V_V}{V}$$

with V_S the volume of the matrix
V_V the volume of the voids
V the total volume

- distension:

$$\alpha = \frac{\rho_s}{\rho} = \frac{1}{1-\phi}$$

with ρ_s the density of the matrix
ρ the bulk density

$$\alpha_0 \geq \alpha \geq 1$$

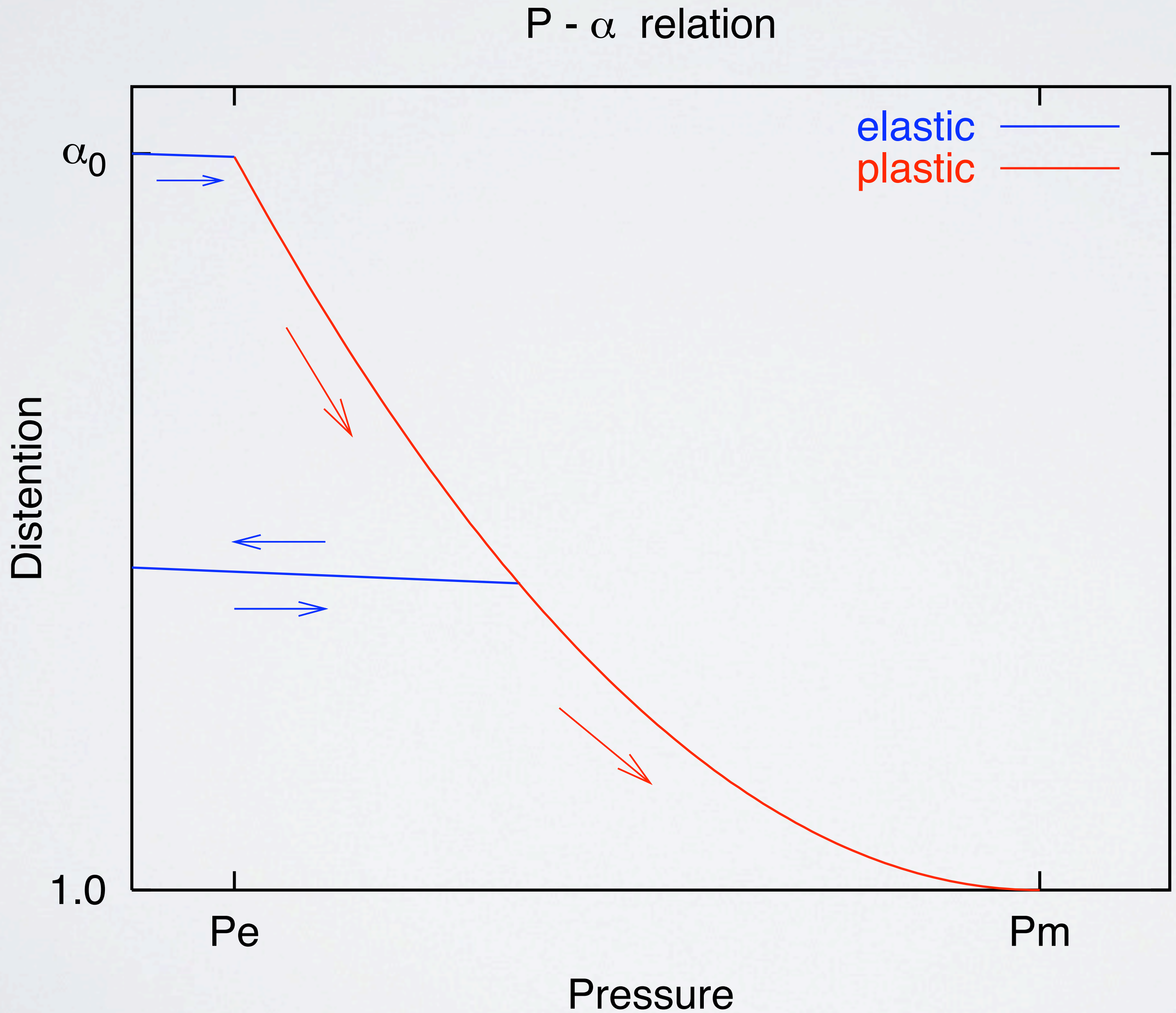
α_0: initial distension

1: completely crushed

Crush curve

Distension is defined as a function of pressure: $\alpha = \alpha(P)$

the so-called $P - \alpha$ *model (Hermann 1969)*



Implementation

- Distension modifies the equation of state (eos):

$$P = \frac{1}{\alpha} P_s(\rho_s, E_s) = \frac{1}{\alpha} P_s(\alpha\rho, E) \qquad \alpha = \frac{\rho_s}{\rho}$$

where $P_s(\rho_s, E_s)$ is the EOS of the solid phase of the material.

Several eos for solid material exist, e.g., Tillotson EOS, ANEOS, etc.

- Time evolution of distension:

$$\dot{\alpha}(t) = \frac{\dot{E}\left(\frac{\partial P_s}{\partial E_s}\right) + \alpha\dot{\rho}\left(\frac{\partial P_s}{\partial \rho_s}\right)}{\alpha + \frac{d\alpha}{dP}\left[P - \rho\left(\frac{\partial P_s}{\partial \rho_s}\right)\right]} \cdot \frac{d\alpha}{dP}$$

Distension: feed-back on the dynamics

- Distention and deviatoric stress
  - Idea: compute the deviatoric stress as a function of the matrix variables

$$[\vec{\nabla}\vec{v}]_s = f[\vec{\nabla}\vec{v}] \quad \rightarrow \quad \frac{dS^{ij}}{dt} \rightarrow f\frac{dS^{ij}}{dt} \quad \text{where} \quad f = 1 + \frac{\dot{\alpha}\rho}{\alpha\dot{\rho}}$$

- Distention and damage
  - Since both damage D and distention **α** are volume ratios, we can relate the two by (linear relation):

$$D = 1 - \frac{(\alpha - 1)}{(\alpha_0 - 1)}$$

$$\boxed{\alpha = 1 \rightarrow D = 1}$$

total damage =

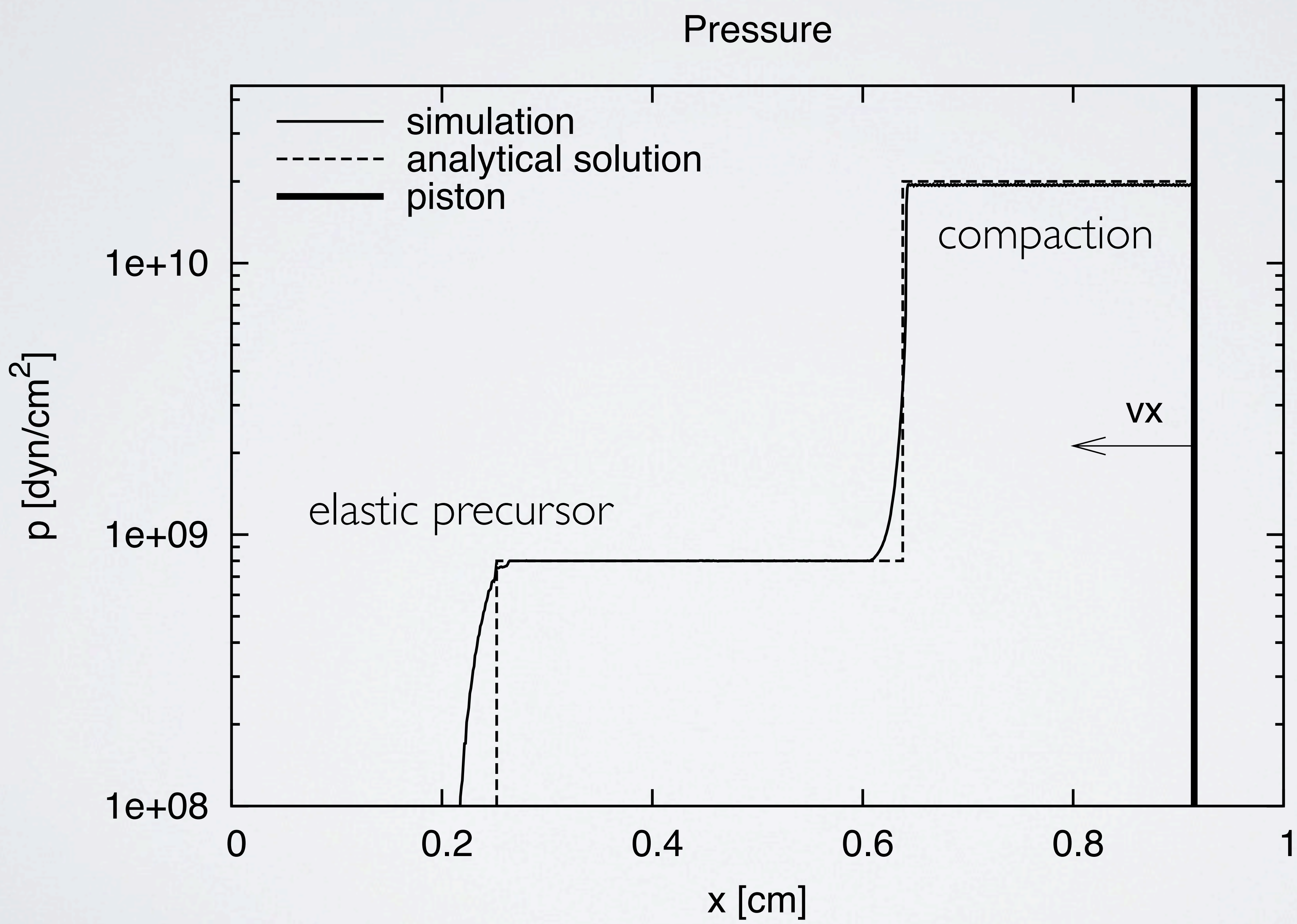
tension damage (Weibull flaws)

+

compression damage (breaking pores)

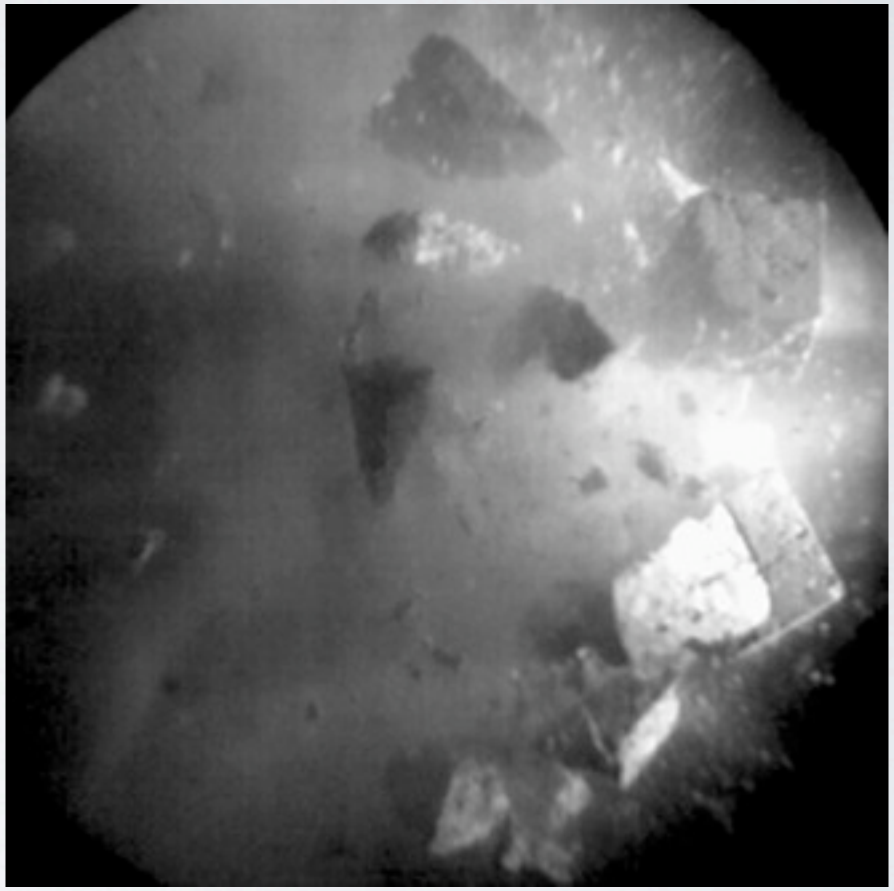
Simple test: Compaction wave

1D pressure wave



Comparison with laboratory experiments

- Experiments by A. Nakamura and K. Hiraoka (Kobe University) using a two stage light gas gun at ISAS
- Impact of nylon/glass projectile on porous pumice
- Measured material properties: Crush-curve, bulk density, tensile strength
- Measured quantities:
    - Mass distribution of fragments
    - Antipodal velocities
    - Snapshots of expanding fragments



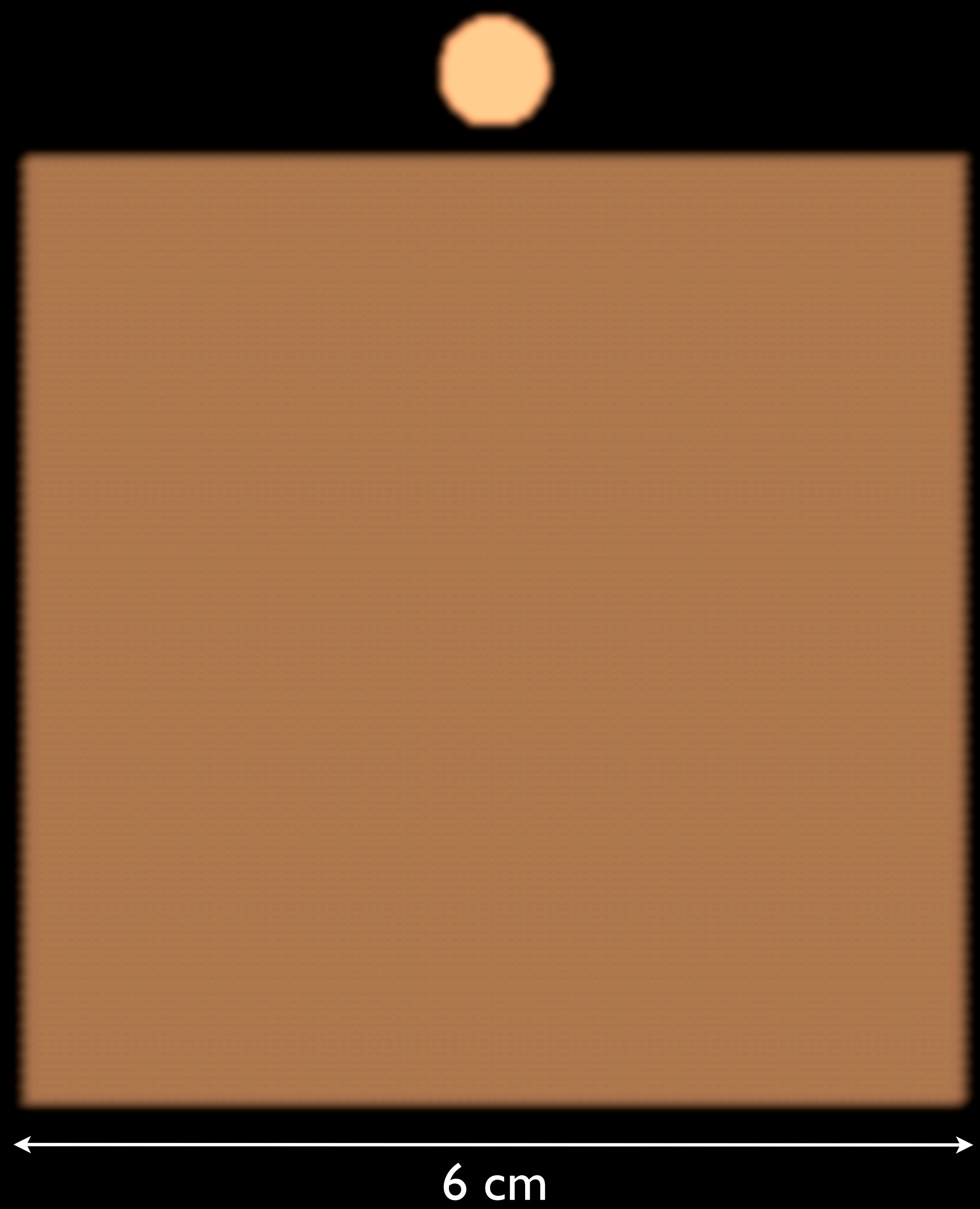
Comparison with laboratory experiments

Simulation of 4 shots:

Projectile					Target	
	Material	Diameter mm	Mass g	Velocity km/s	Mass g	Porosity %
418-4	Nylon	7	0.21	2.58	147.8	71
824-6	Glass	3.2	0.04	4.47	40.1	73
825-4	Nylon	7	0.21	3.28	38.7	75
70427	Nylon	3.2	0.02	3.94	37.3	75

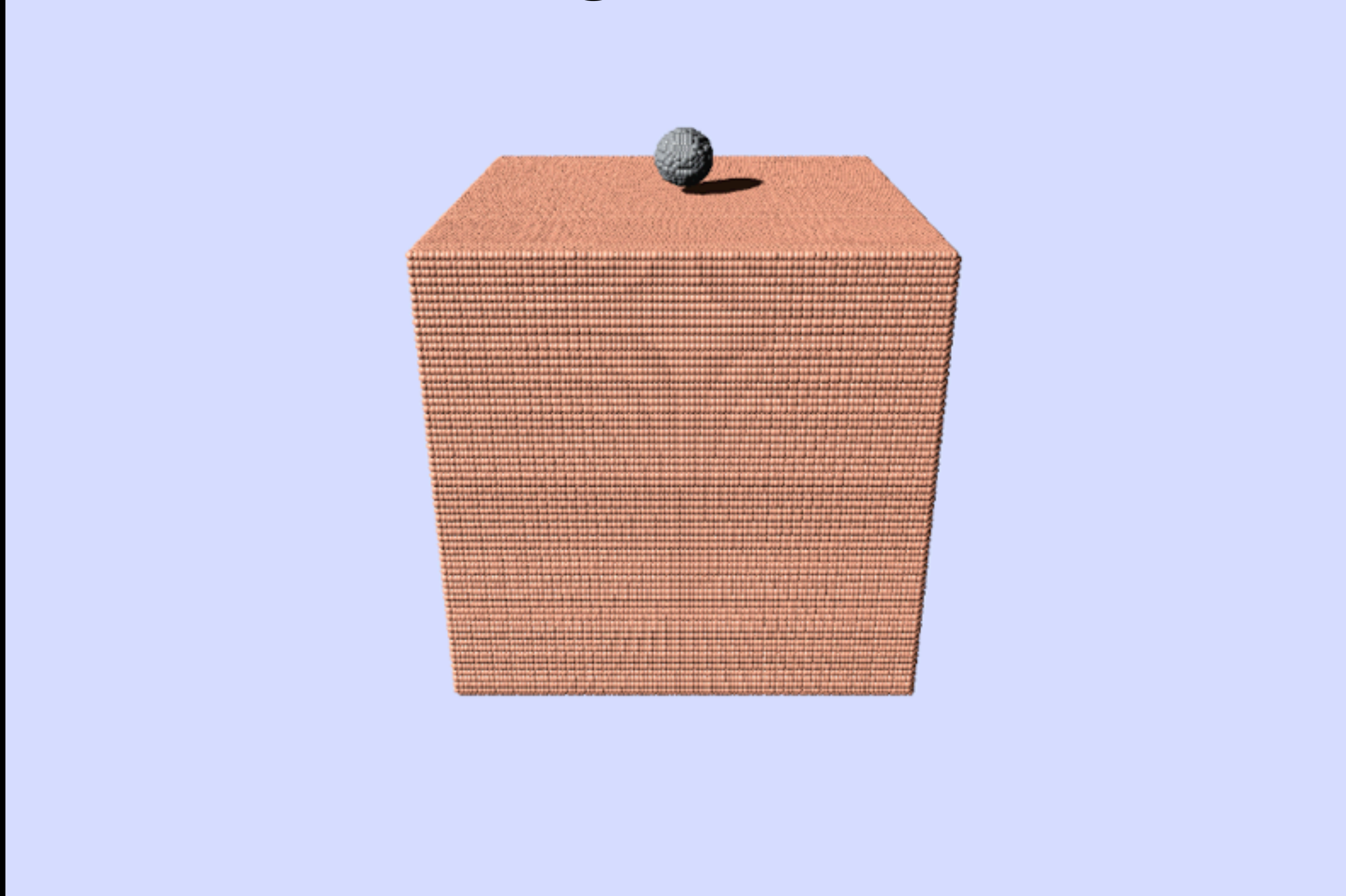
Same material parameters (the measured ones) for all simulations

Number of particles: $1.4\ 10^6$



Evolution of damage
0 - 0.1 ms

Impact and subsequent expansion of fragments



Impact: 0 - 200 ms
Expansion: 0.2 ms - 16 ms

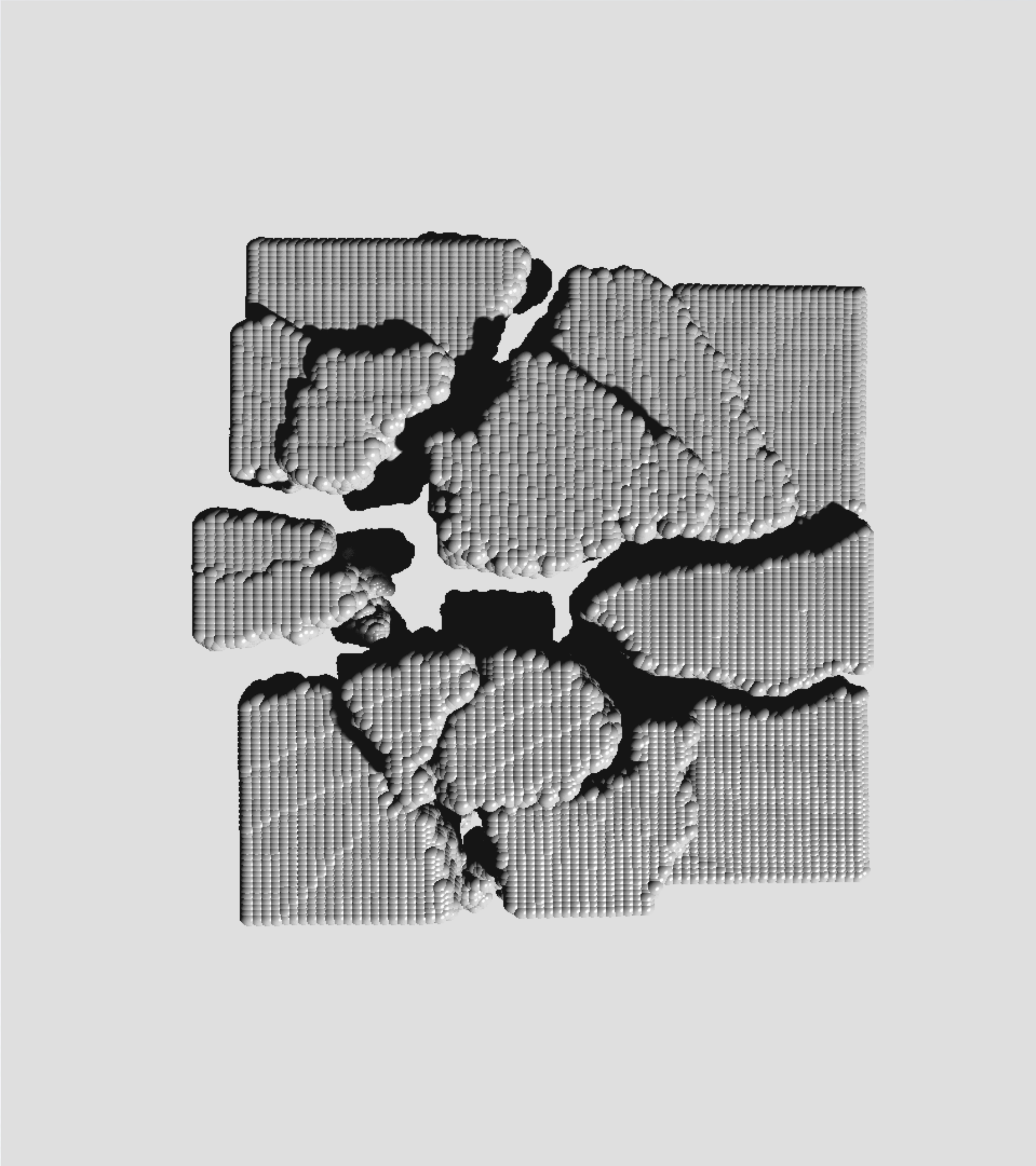
Comparison with laboratory experiments

t = 1.5 ms

Experiment

$v_{antipodal}$ = 5.9±1.6 m/s

Simulation (porous)

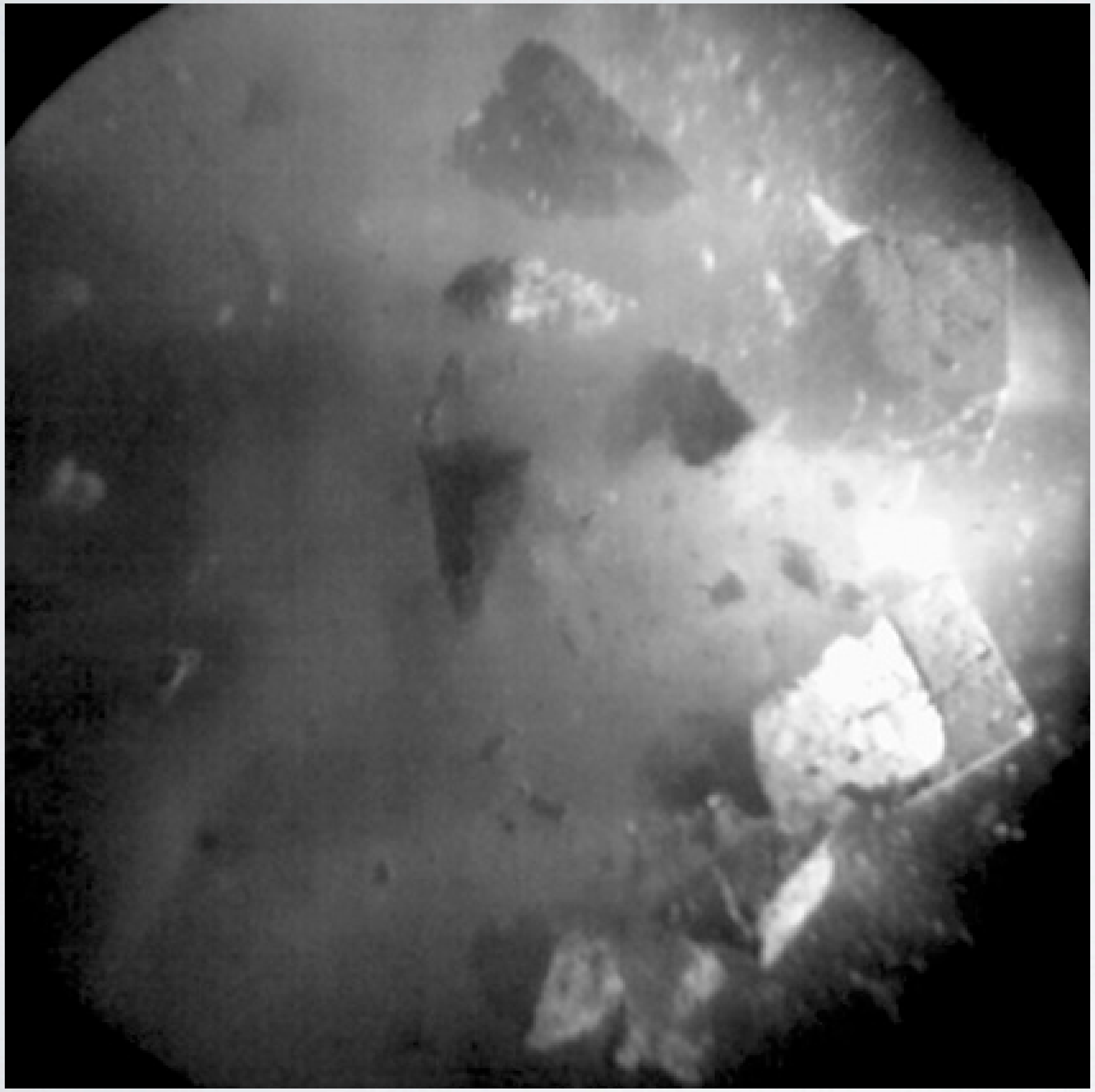


$v_{antipodal}$ = 5.6 m/s

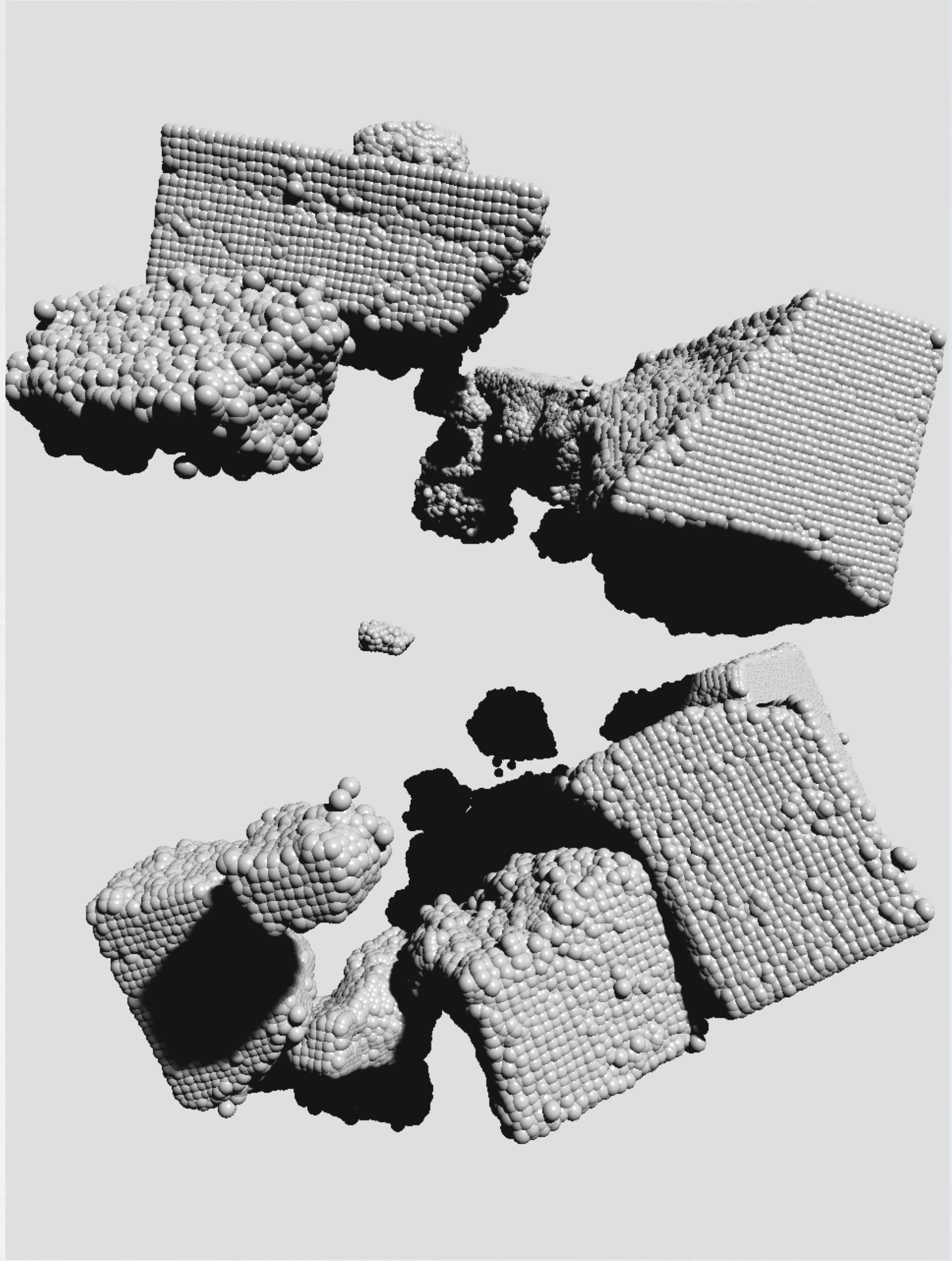
Comparison with laboratory experiments

t = 8.0 ms

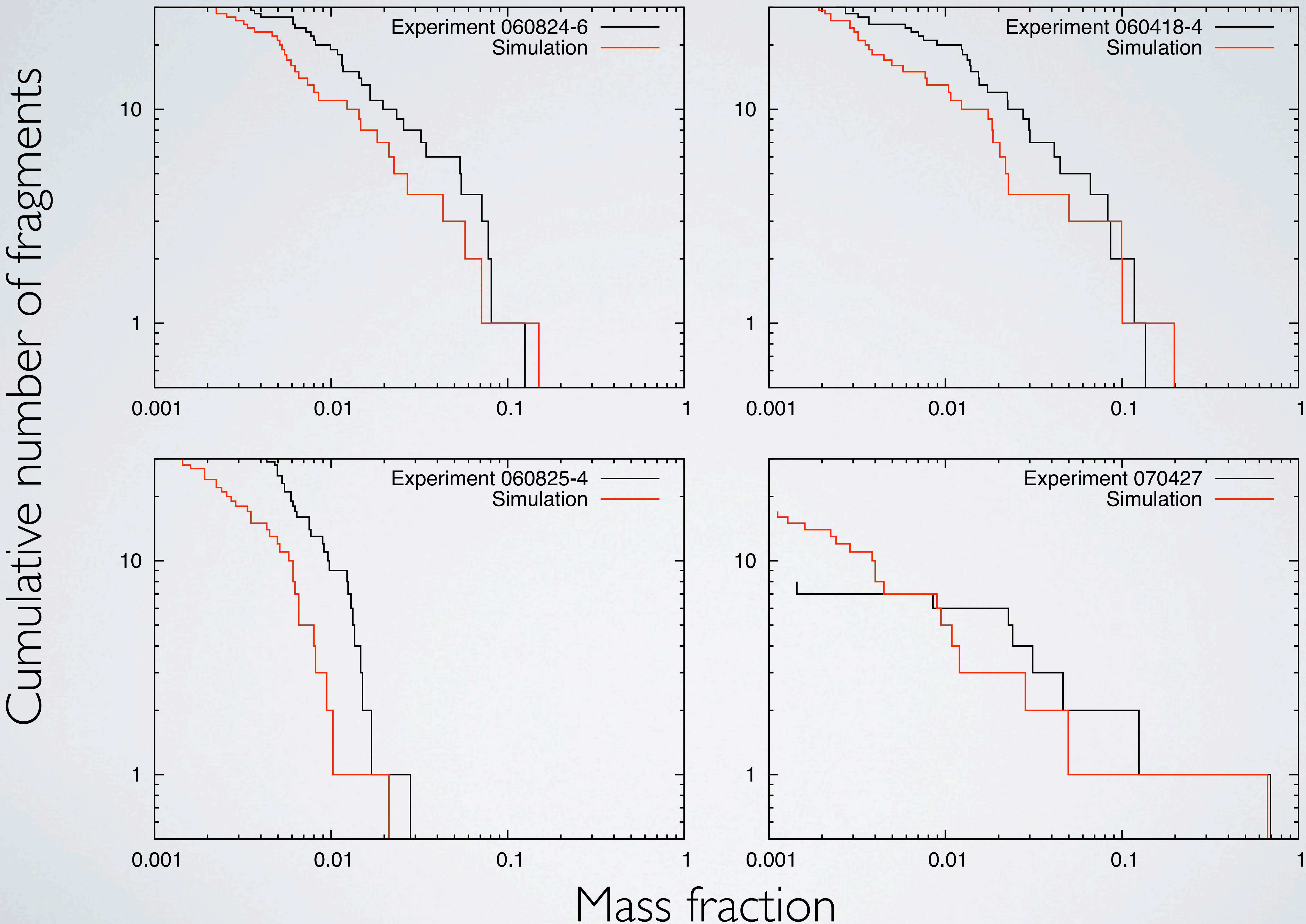
Experiment



Simulation (porous)

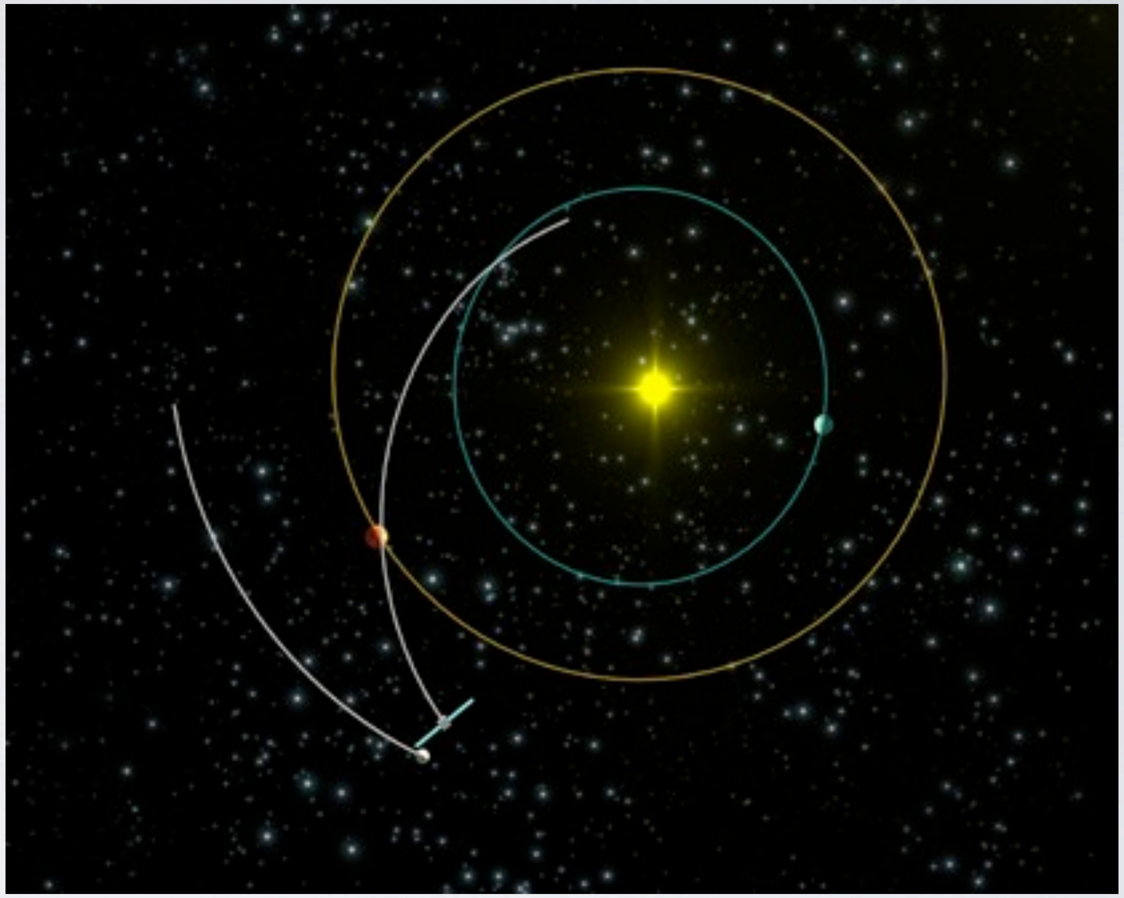


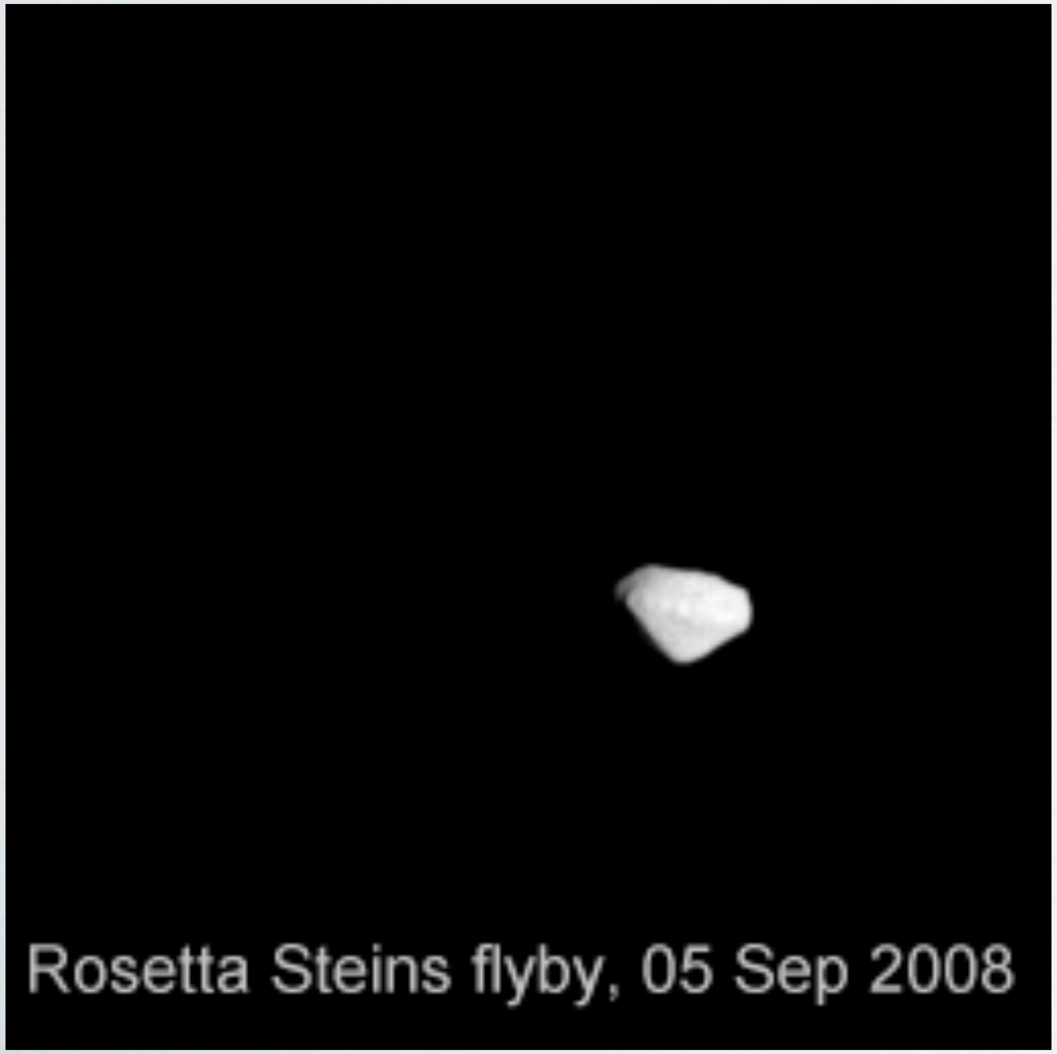
Cumulative mass distribution

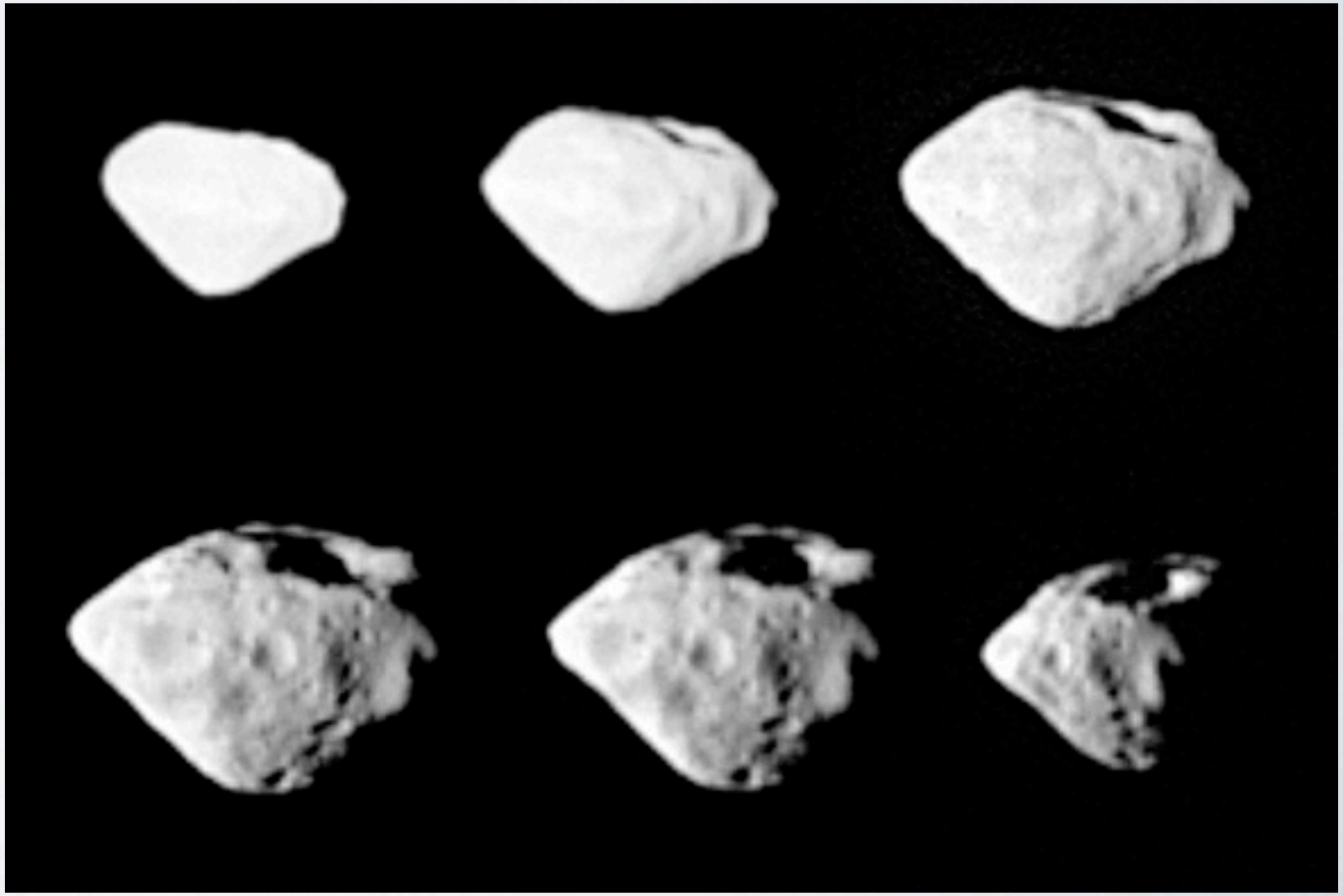


Asteroid Steins

Rosetta Fly-by
on 5 September 2008





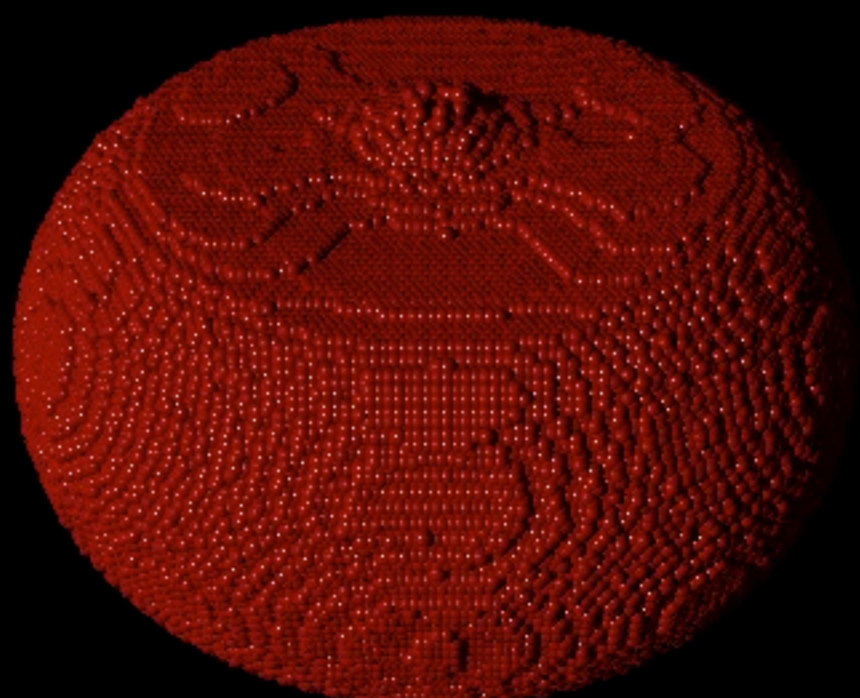


Asteroid Steins

What kind of impact produces such a crater?

5.73 x 4.95 x 4.58 km

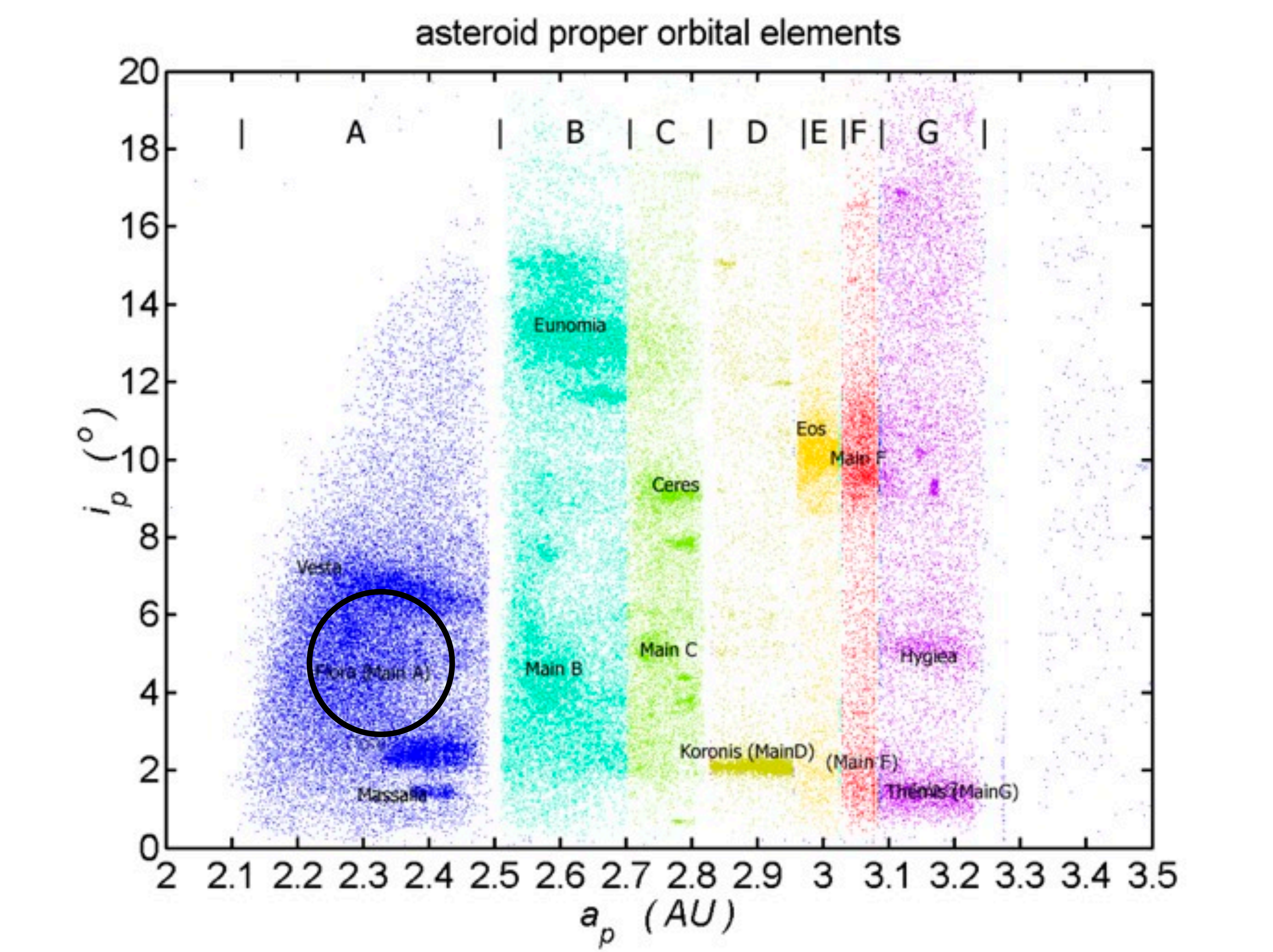
Impactor:
D = 180 m
V_{imp}= 5 km/s



non-porous

porous
(25 % porosity)

Asteroid families: Evidences for distruptive collisions



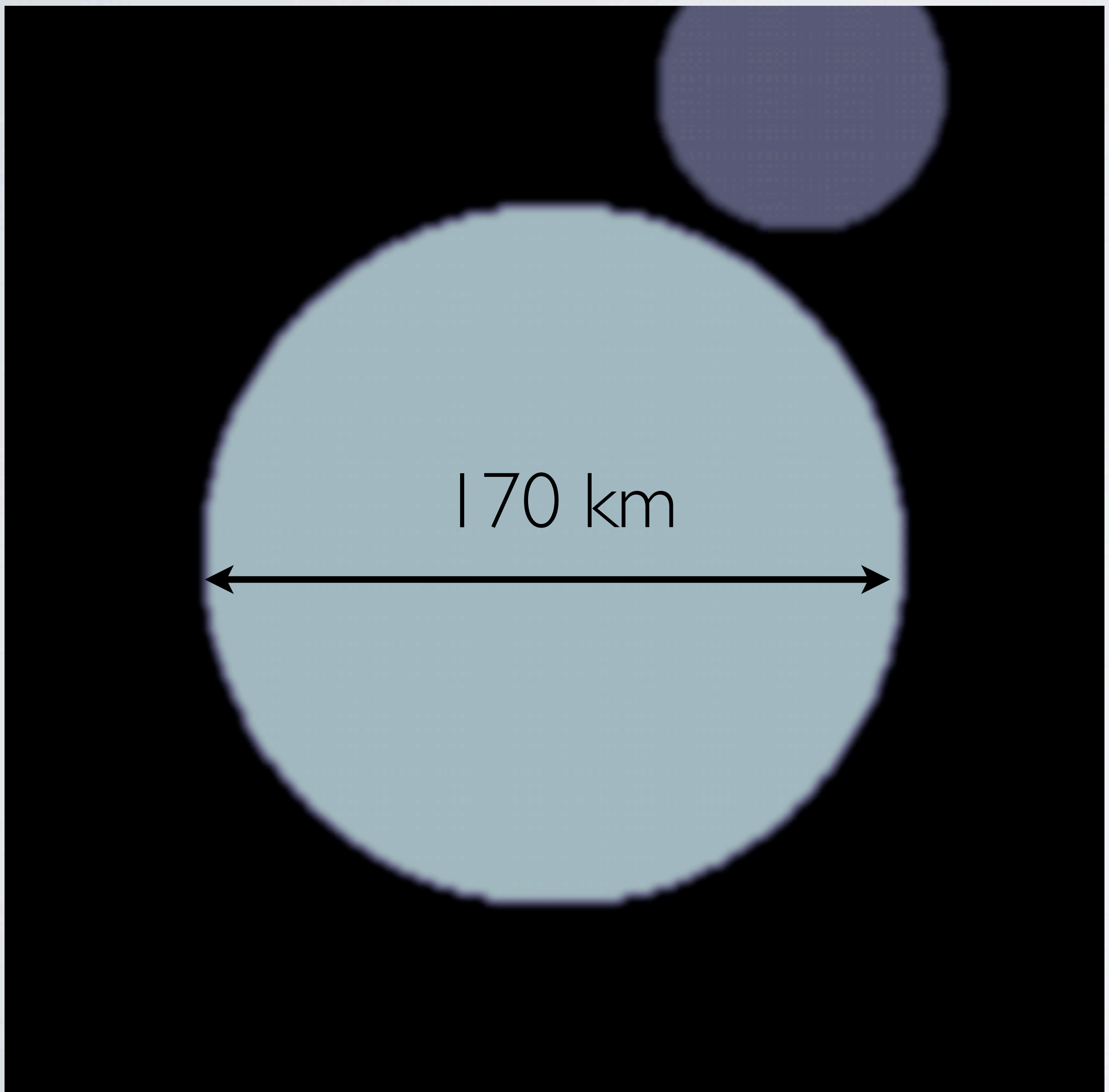
asteroid families provide the laboratory to test codes in the gravitational regime

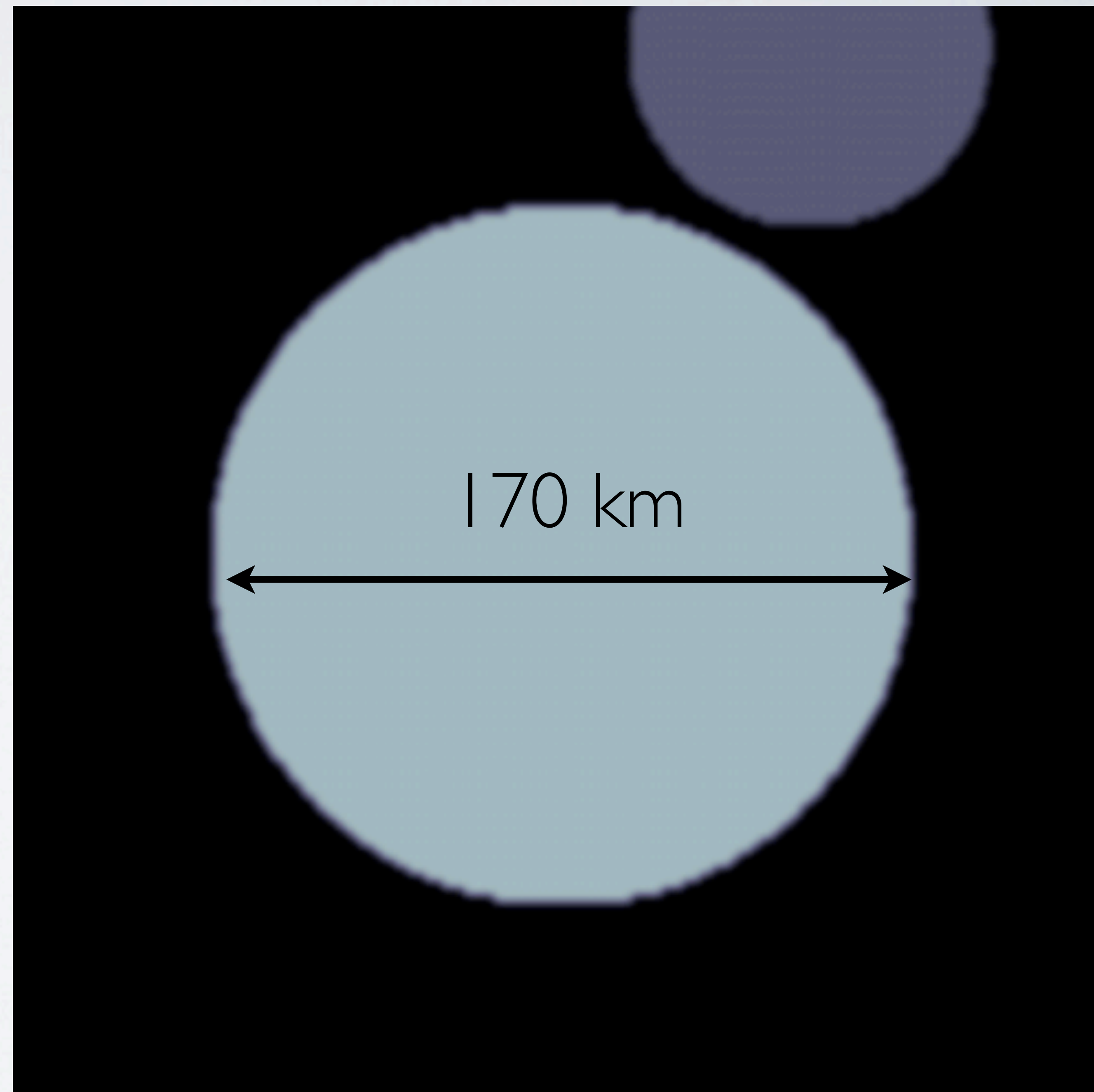
Disruptive collisions (at 3km/s)

Porous

Simulation: t = 0 - 200 s

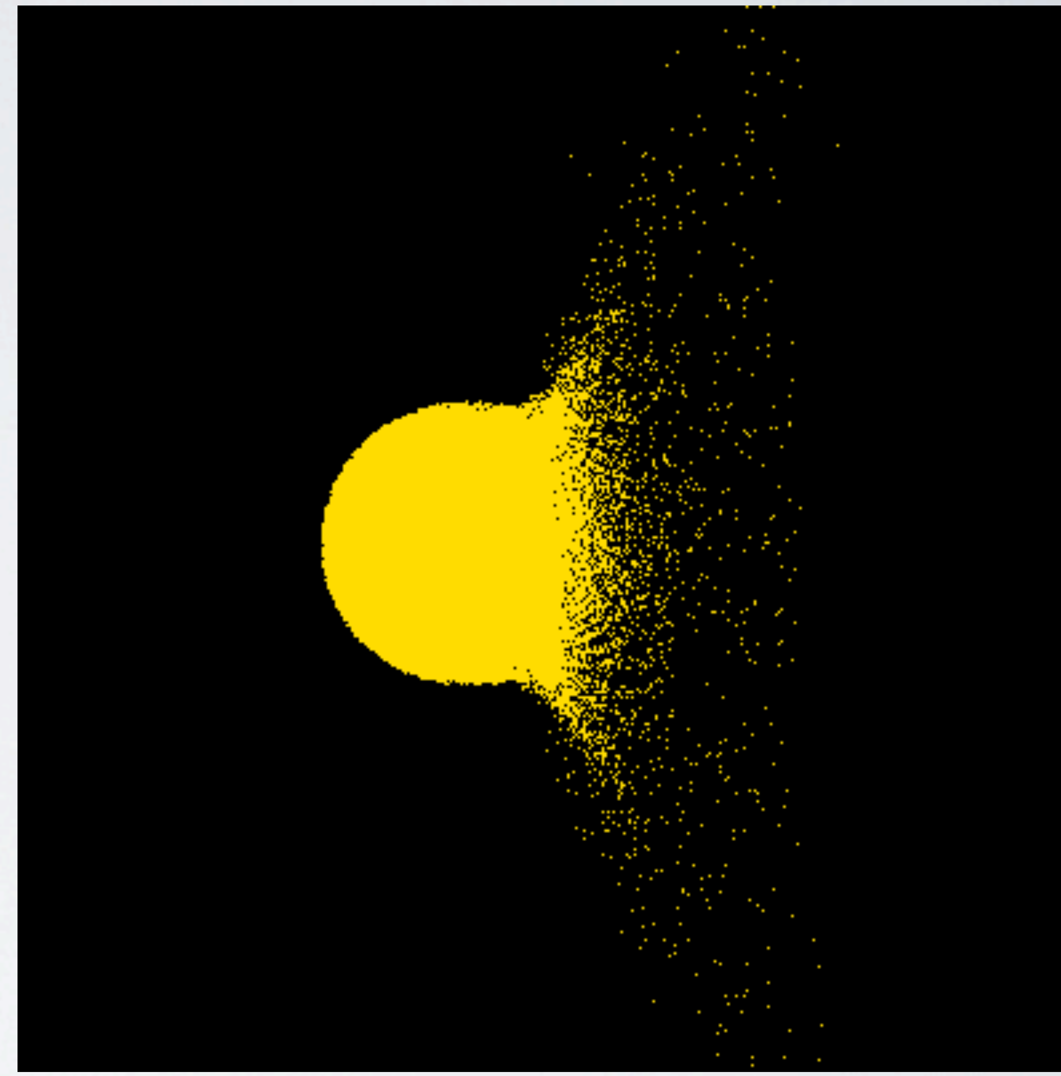
Non-Porous





Families: Disruption and re-accumulation

Flora family: Parent body 164 km



1) the parent body is totally disrupted by a catastrophic impact

SPH simulations

2) expanding debris are re-accumulating to form family members

Collisional N-body simulations

Explain the rubble pile nature of the larger objects

Conclusions

1) Collisions are at the heart of the evolution of the small bodies in the solar system

2) The Lagrangian nature of SPH makes it an ideal tool to simulate impacts and collisions

3) Material parameters play a key role in the modeling
   - can/should be measured in laboratory experiments
   - are not known for most of the bodies.

→ *inverse problem: Use collisions to probe the material properties...*