Keynote lecture
2017 SPHERIC Beijing International Workshop

Smoothed Particle Hydrodynamics, fact checking: from theory to applications

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For a start: SPH in 3 words

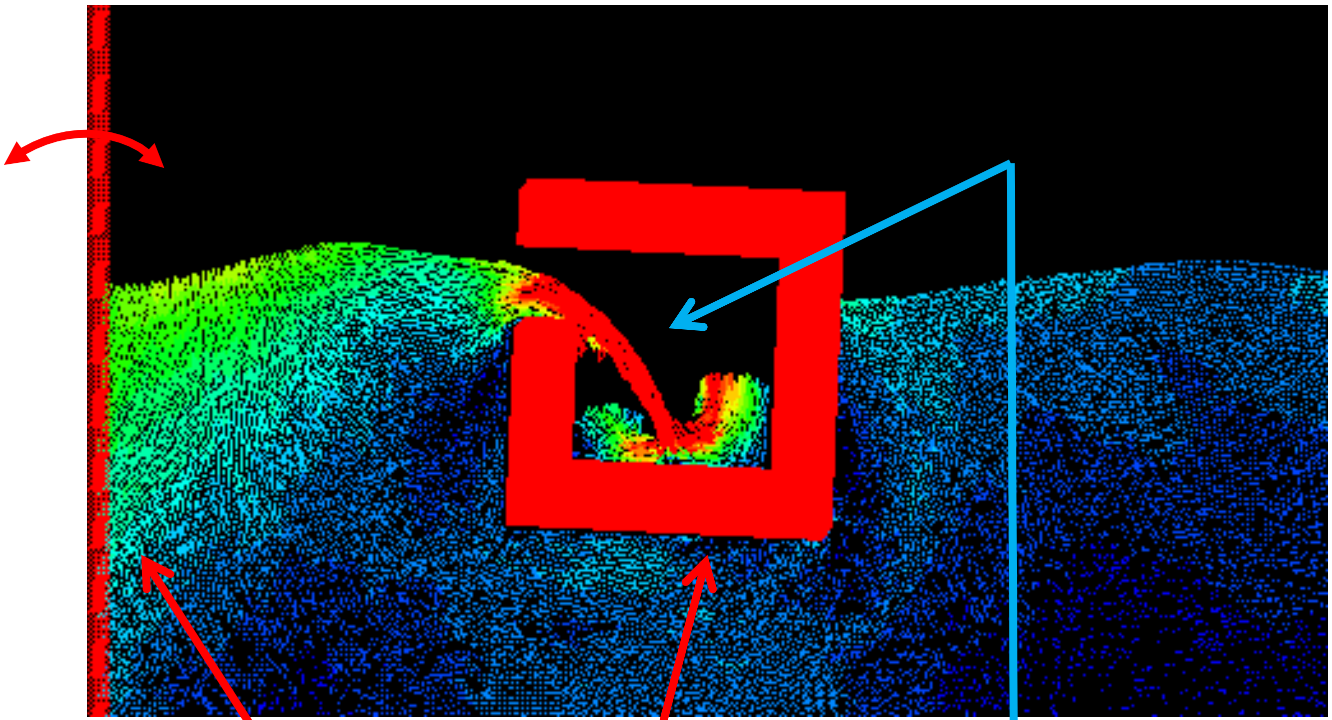
Smoothed Particle Hydrodynamics

A computational method for solving continuum mechanics problems…

… with large deformations, multiple objects and complex interfaces

Large deformations and complex interfaces…

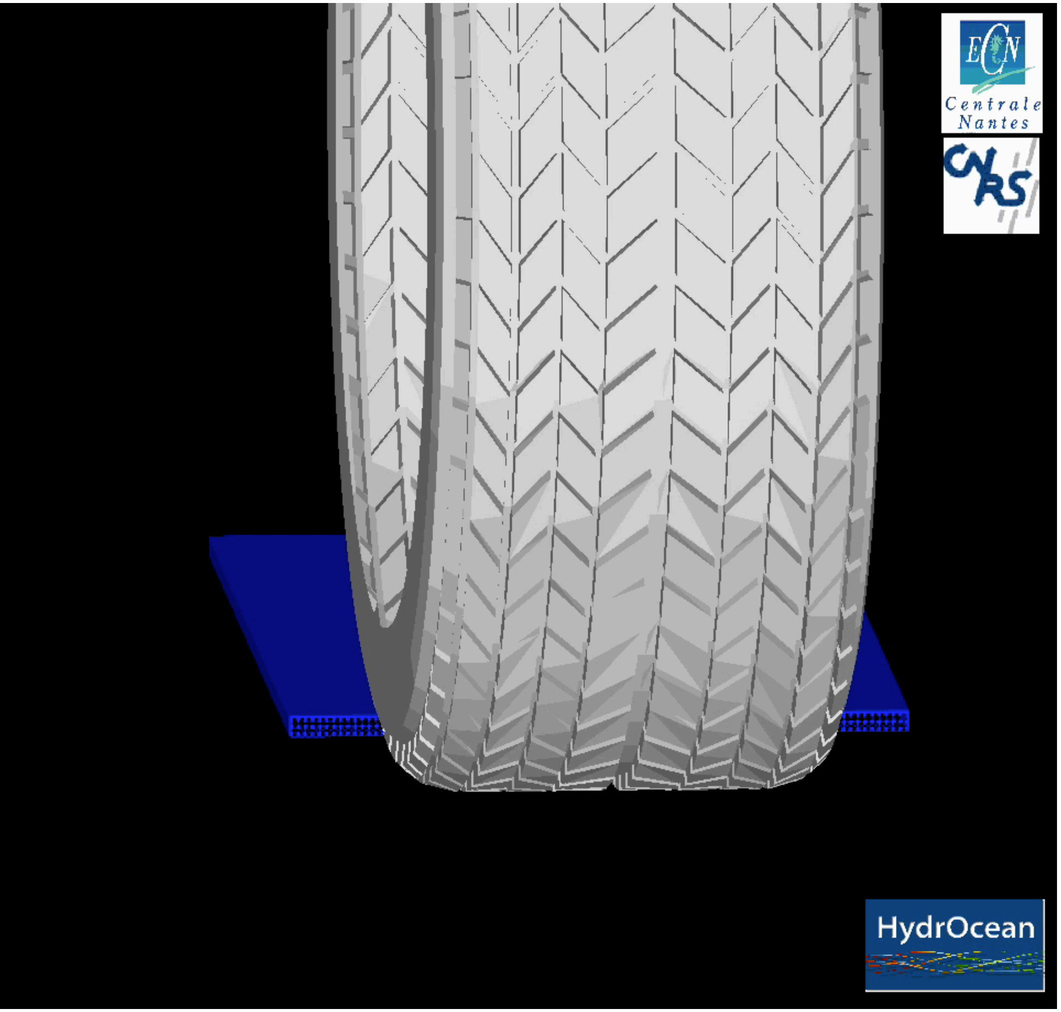
What we were doing in 2000



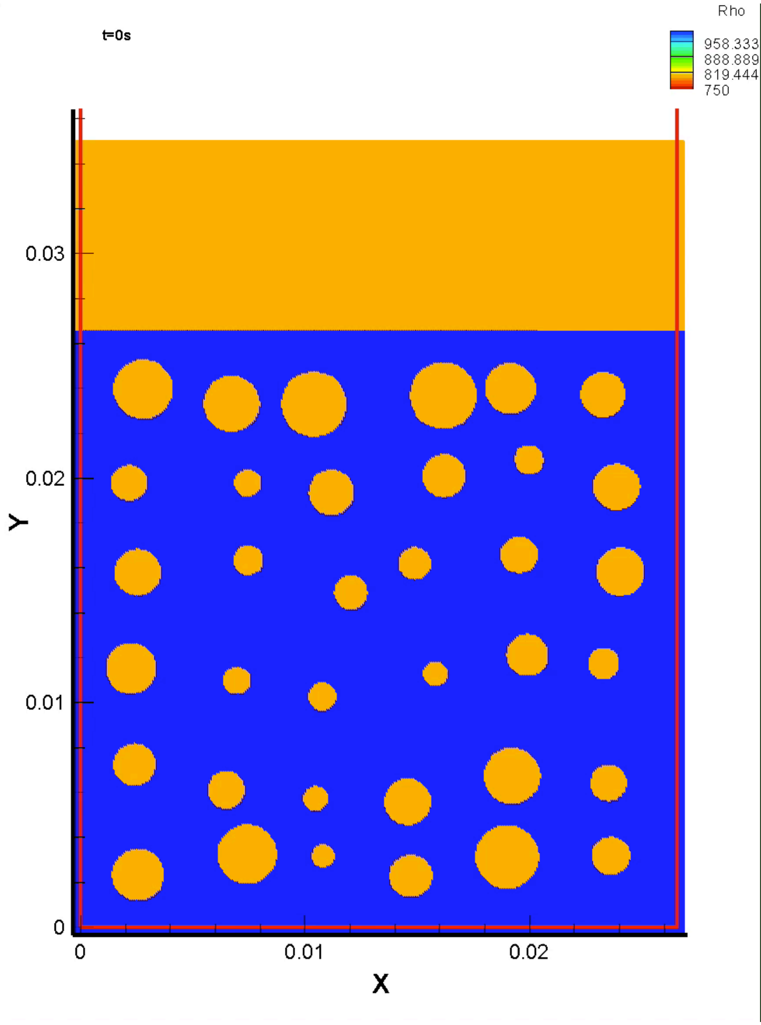
2 moving bodies = wavemaker + floating body + 1 complex interface

Large deformations and complex interfaces…

What we were doing 10 years ago



Tire aquaplanning



Water-oil separation

Large deformations and complex interfaces…

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What we were doing 5 years ago

HYDROCEAN

ECN Centrale Nantes

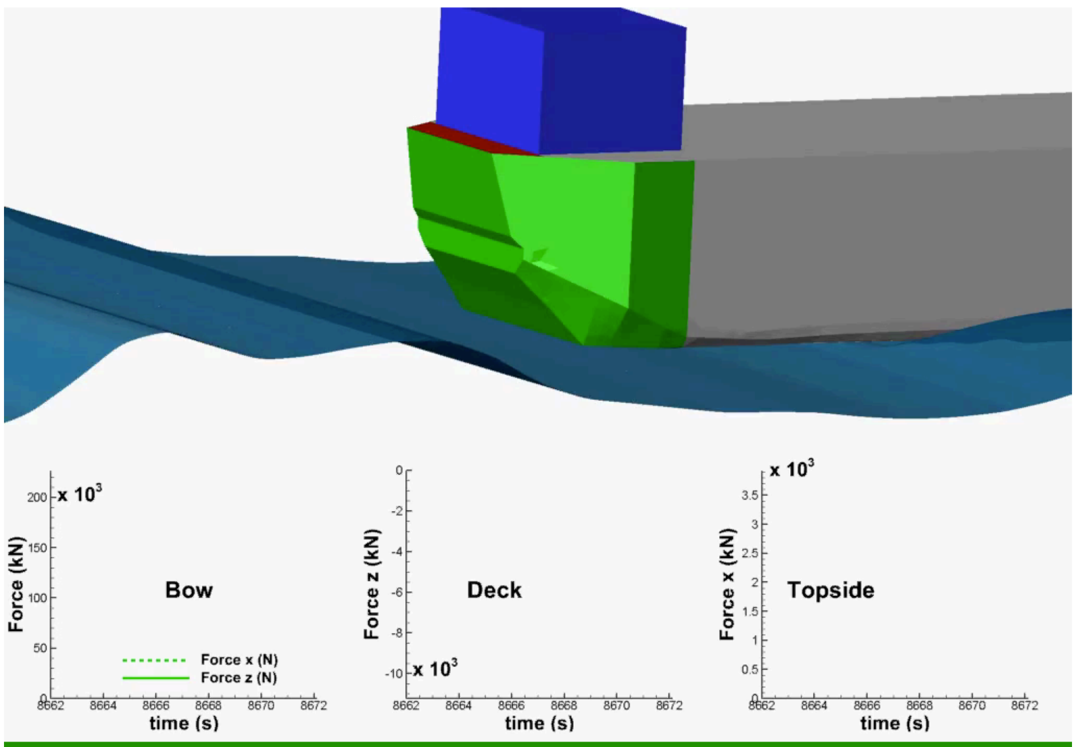
SPH–flow

Focused wave on a FLNG

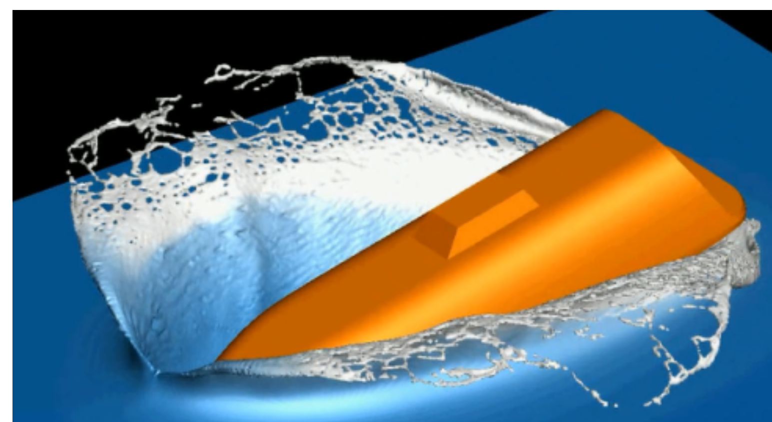
- 20 500 000 particles
- 2048 cpus

TOTAL

Technip



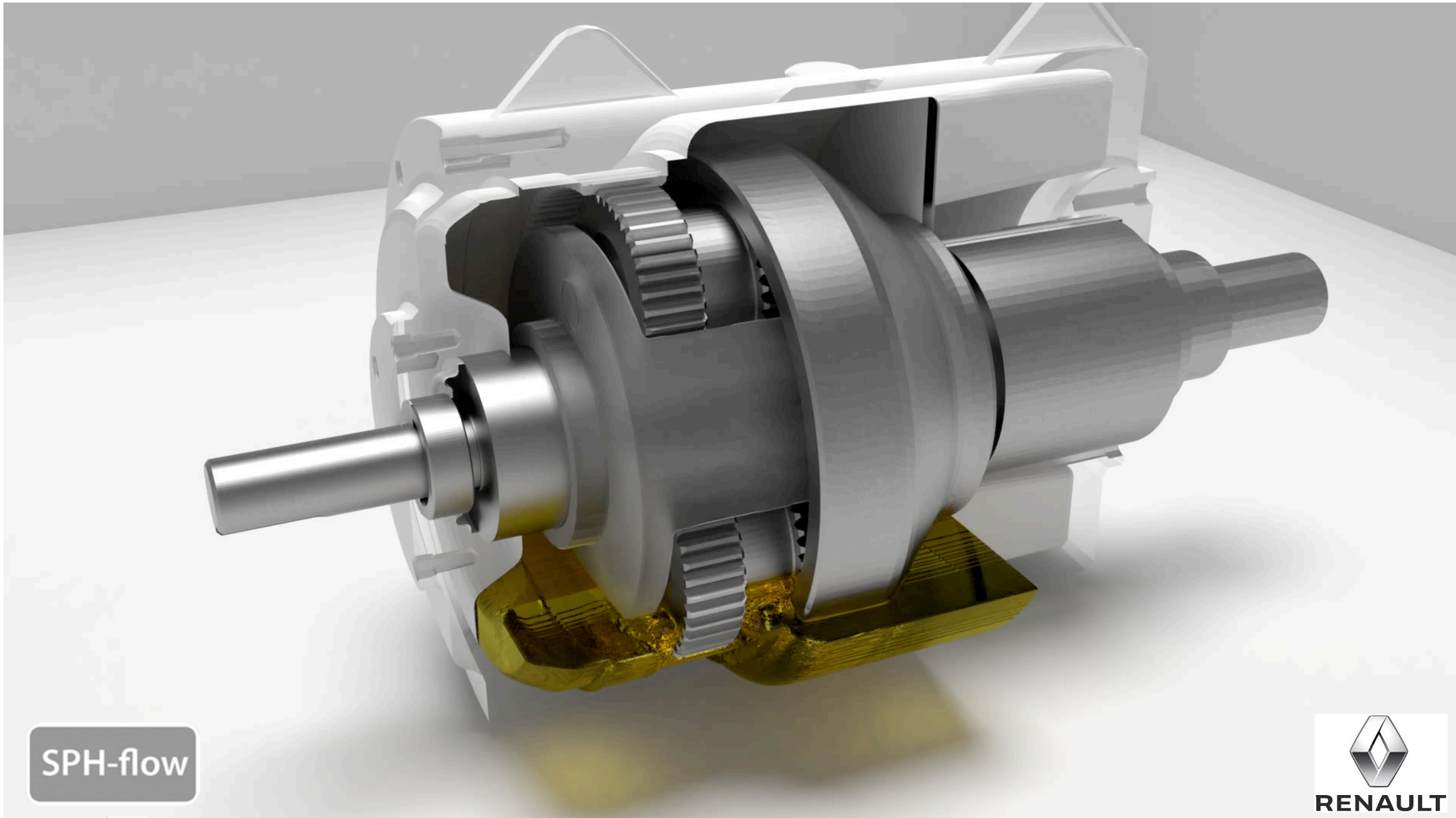
Greenwater loads on a ship deck



Lifeboat emergency ditching

Large deformations and complex interfaces…

What we are doing now



NEXTFLOW CENTRALE NANTES + Marine Technology Research Institute INSEAN

Gearbox lubrification

For a start: SPH in 3 words

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Smoothed Particle Hydrodynamics

↓

Complex interfaces = large deformations, fragmentation, coalescence…

+

Complex (multiple body) motions = small gaps between objects, contact…

=

Very difficult for mesh-based methods, especially if we want accurate results

$\Rightarrow$ follow the interfaces/motions = Lagrangian

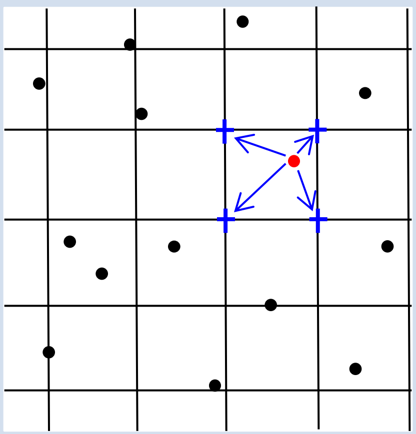
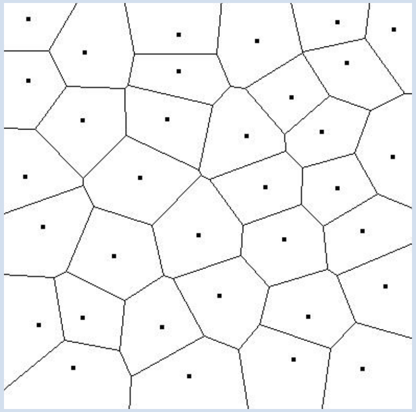
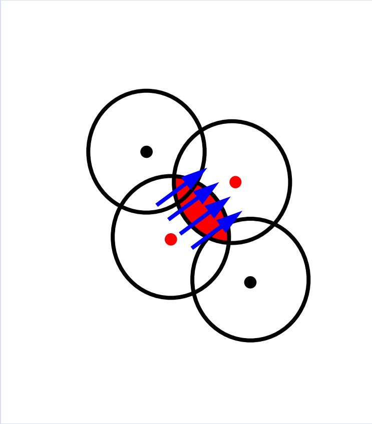
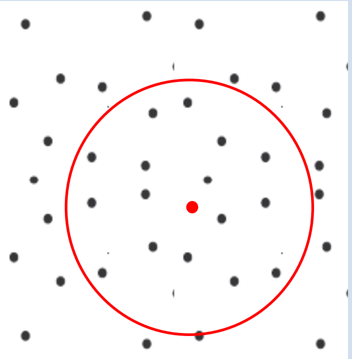
+

$\Rightarrow$ not possible to use a mesh = meshless

=

particle method

Meshless?

Projection	Tesselation	Face construction	Truly meshless
			
Particle-Mesh …	Voronoi-FVM Particle-FEM …	FVPM …	SPH MPS …

Meshless?

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Truly meshless

+ : any configuration can be easily described

- : no description of how the volume of a particle is spread around its location, and on how it will deform in time

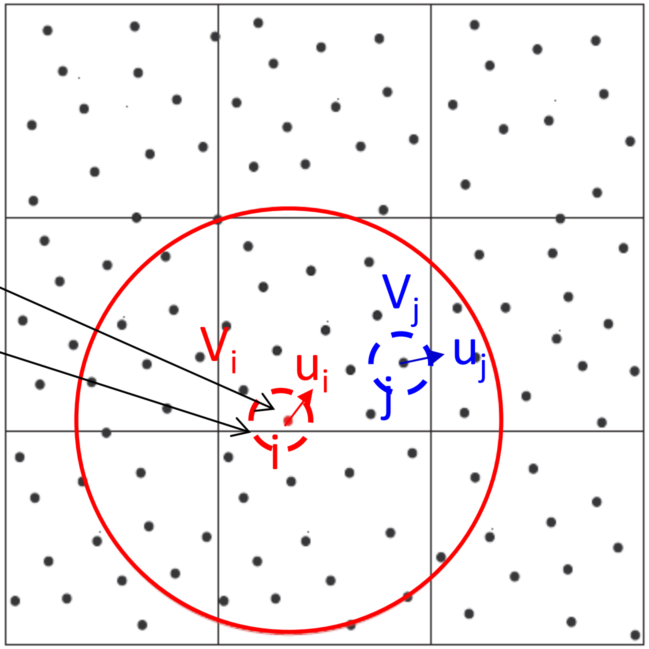
Partly meshless (projection/reconstruction)

+ : easier to define convergent operators / make mathematical analysis of the schemes

\- : less general / complex and costly implementation / **how to treat interfaces? especially free-surface?**

Particle

Particle: a volumic element (volume V_i) of barycenter the scattered point *i*



Particle method:

=

meshless (i.e. no connectivity, NOT no space discretization)

+

Lagrangian (material evolution : d·/dt = ...): particle *i* evolves at its material speed $\mathbf{u}_i$

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Smoothed Particle Hydrodynamics

E.g., for Navier-Stokes, in Lagrangian (material) formalism

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}$$

$$\frac{d\rho}{dt} = -\rho \, \mathrm{div}\mathbf{u}$$

$$\frac{d\mathbf{u}}{dt} = \mathbf{g} - \frac{\mathrm{grad}p}{\rho} + \nu\Delta\mathbf{u} + \frac{\nu}{3}\mathrm{grad}(\mathrm{div}\mathbf{u})$$

$$p = f(\rho)$$

=> how do we calculate the spatial operators with no mesh?

For a start: SPH in 3 slides

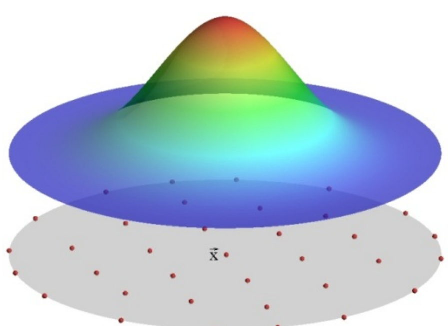
=> mollification (1) + discretisation (2)

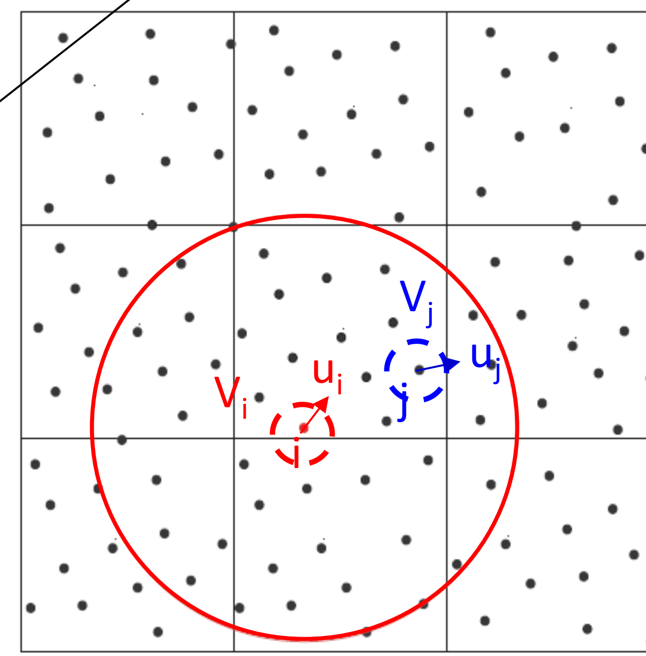
$$< \mathbf{u} >_i = \int \mathbf{u}(x)\delta(|x-x_i|)\,dx \overset{1}{\cong} \int \mathbf{u}(x)W(|x-x_i|)\,dx \overset{2}{\cong} \sum_j \mathbf{u}_j W(|x_j-x_i|)\,V_j$$

Known at the neighbour scattered points (particle)

Volume associated to the scattered points (particles)

Analytical function (kernel)





Compact support!!!

+ transfer of the differentiation from the field to the kernel to get differential operators:

analytical

$$\nabla f(\vec{r}) \overset{1}{\approx} \int_D \nabla f(\vec{x})W(\vec{r}-\vec{x})d\vec{x} = \int_D f(\vec{x})\nabla W(\vec{r}-\vec{x})d\vec{x}$$

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Smoothed Particle Hydrodynamics

E.g., for Navier-Stokes, in Lagrangian (material) formalism

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$$p = f(\rho)$$

Explicit time integration

For a start: SPH in 3 slides

Smoothed Particle Hydrodynamics

A computational method for solving continuum mechanics problems…

… with large deformations, multiple objects and complex interfaces

⇒ we are doing engineering, not movies or games

⇒ we want accurate stresses (e.g. pressure), forces, deformations…

SPH fact checking

partly true

1. SPH cannot be solved implicitly (as mesh-based) / it has very small time steps

1. Fully-implicit? Small time steps?

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A fully implicit scheme would mean that time derivatives are expressed using the solution at next time step (n+1), including the displacements:

$$\frac{d\vec{x}}{dt} = \vec{V}^{(n+1)}$$

⇒ This would lead to a complex implementation in practice, and attempts we made showed that the resulting scheme is too diffusive

⇒ No one does that in SPH

⇒ 2 ways : fully-explicit OR, at best, semi-implicit

⇒ Fully explicit = WCSPH

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}$$

$$\frac{d\rho}{dt} = -\rho \mathrm{div}\mathbf{u}$$

$$\frac{d\mathbf{u}}{dt} = \mathbf{g} - \frac{\mathrm{grad}p}{\rho} + \nu\Delta\mathbf{u} + \frac{\nu}{3}\mathrm{grad}(\mathrm{div}\mathbf{u})$$

$$p = f(\rho)$$

⇒ Stability criteria: $\Delta t < \Delta x / c_s$

Semi-implicit = ISPH (or MPS)

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}$$

$$\mathrm{div}\mathbf{u} = 0$$

$$\frac{d\mathbf{u}}{dt} = \mathbf{g} - \frac{\mathrm{grad}p}{\rho} + \nu\Delta\mathbf{u}$$

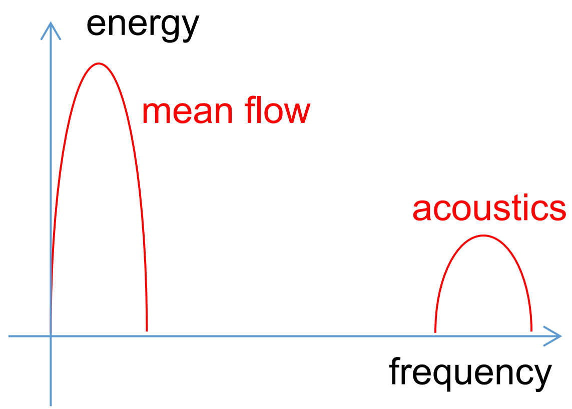
$$\Delta t < \Delta x / |\mathbf{u}|_{max}$$

SPH fact checking

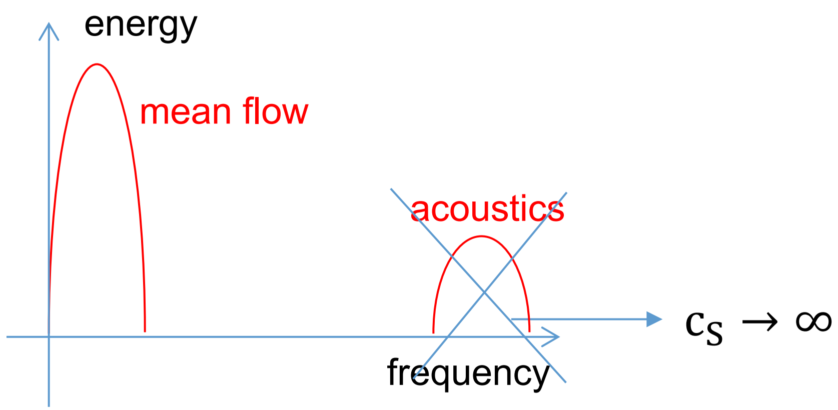
partly true	1.	SPH cannot be solved implicitly (as mesh-based) / it has very small time steps
partly true	2.	Weak-compressbility is an unphysical trick

2. Weak-compressbility is an unphysical trick

Low Mach physical situation = acoustics is superimposed to the incompressible part of the flow, and fully separated in frequencies

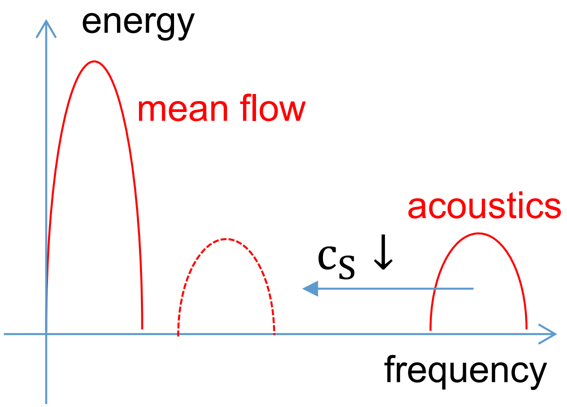


=> Incompressible assumption: all waves have infinite speed

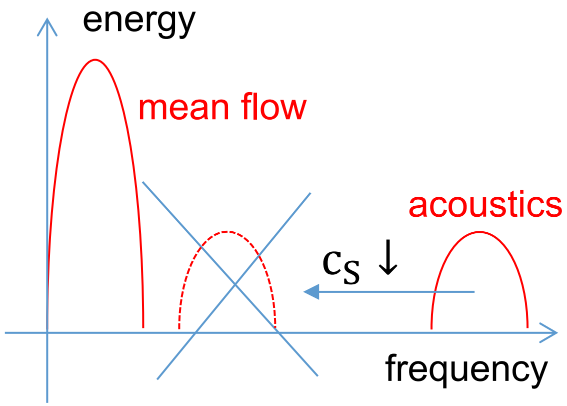


2. Weak-compressbility is an unphysical trick

=> Weakly-compressible assumption: we can change the sound speed since we are not interested in the acoustic part of the flow (at low Mach)



=> N.B. : Weakly-compressible assumption + filtering



2. Weak-compressbility is an unphysical trick

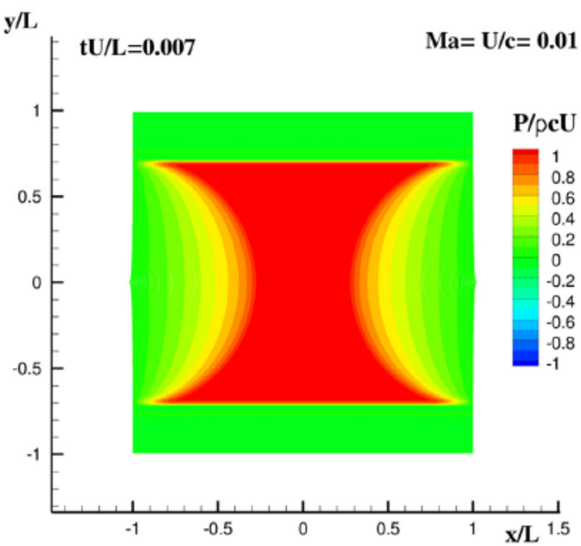
⇒ Weakly-compressible assumption, summarizing:

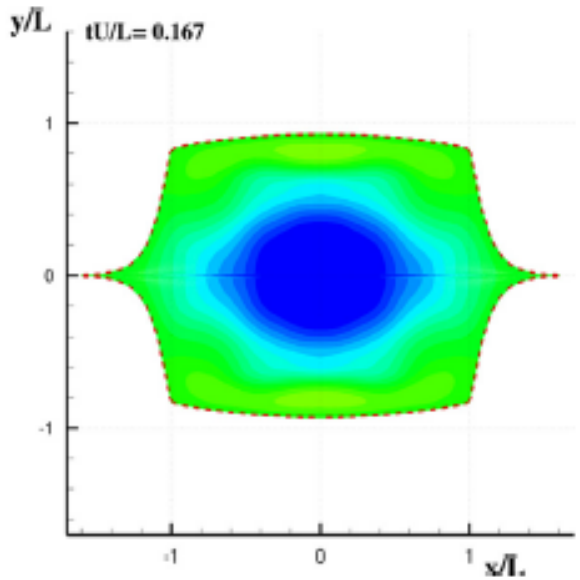
- It is not a bigger assumption than supposing the flow incompressible
- Without fiterting it implies that physical pressure oscillations should be present in the solution (at unphysical frequencies)
- With perfect filtering, it is equivalent to the incompressible assumption
- It permits to lower the sound speed provided Ma stays lower than 0.1 at least, i.e. $c_s > 10|\mathbf{u}|_{max}$
- It thus permits to loosen the acoustic CFL stability condition: $\Delta t < \frac{\Delta x}{(10|\mathbf{u}|_{max})}$
- This induces only a factor 10 with ISPH, $\Delta t < \Delta x / |\mathbf{u}|_{max}$

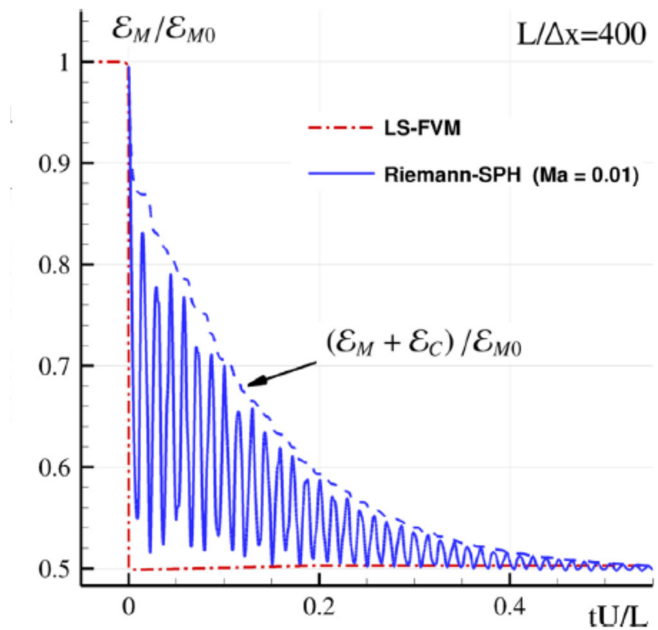
(N.B. : ISPH requires imposing conditions at the free surface + solving a system at each time step)

2. Weak-compressbility is an unphysical trick

=> we checked that weakly-compressible solution matches theoretical incompressible one once acoustic oscillations are damped, even at impact







Le Touzé D. et al., A critical investigation of smoothed particle hydrodynamics applied to problems with free-surfaces, Int. J. Numer. Meth. Fluids **73**, 2013

Marrone S. et al., Prediction of energy losses in water impacts using incompressible and weakly compressible models, J. Fluid Struct. **54**, 2015

SPH fact checking

partly true	1.	SPH cannot be solved implicitly (as mesh-based) / it has very small time steps
partly true	2.	Weak-compressbility is an unphysical trick
not fully true	3.	SPH conserves everything

3. SPH conserves everything

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Let's restrict to Euler equations for now (perfect fluid)

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}$$

$$\frac{d\rho}{dt} = -\rho\,\mathrm{div}\mathbf{u}$$

$$\frac{d\mathbf{u}}{dt} = \mathbf{g} - \frac{\mathrm{grad}p}{\rho}$$

$$p = f(\rho)$$

for **Momentum** conservation (action/reaction)

$$< \nabla p >_i = \sum_j (p_j + p_i) W(|x_j - x_i|)\, V_j$$

$$< \mathrm{div}\mathbf{u} >_i = \sum_j (\mathbf{u}_j - \mathbf{u}_j).\, W(|x_j - x_i|)\, V_j$$

for **Energy** conservation (Hamiltonian)

Mass is conserved (particle method)

Total volume is not conserved!

Colagrossi A. et al., Theoretical considerations on the free-surface role in the smoothed-particle-hydrodynamics model, Phys. Rev. E **79**, 2009

3. SPH conserves everything

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Smoothed Particle Hydrodynamics

Mollification

Meshless Lagrangian method

Continuum mechanics

Vision 1 (original) = Hamiltonian mechanics of a discrete system

A fluid dynamics problem numerically seen as a system of particles using a mollification.

AND

A meshless numerical method for discretizing Lagrangian PDEs.

Vision 2 (standard) = numerical methods for PDEs describing continuous media

SPH fact checking

partly true	1.	SPH cannot be solved implicitly (as mesh-based) / it has very small time steps
partly true	2.	Weak-compressbility is an unphysical trick
not fully true	3.	SPH conserves everything
no more true	4.	SPH is unstable / SPH pressures are noisy

4. Stability? Pressure field quality?

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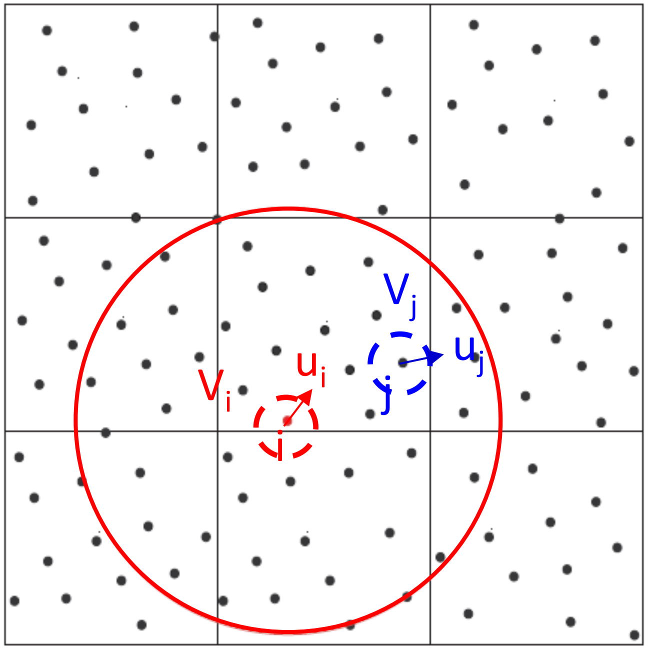
Let’s restrict to Euler equations for now (perfect fluid)

$$\frac{d\mathbf{x}}{dt}=\mathbf{u}$$

$$\frac{d\rho}{dt}=-\rho \mathrm{div}\mathbf{u}$$

$$\frac{d\mathbf{u}}{dt}=\mathbf{g}-\frac{\mathrm{grad}p}{\rho}$$

$$p=f(\rho)$$



$$<\nabla p>_i=\sum_j (p_j+p_i) W(|x_j-x_i|)\, V_j$$

$$<\mathrm{div}\mathbf{u}>_i=\sum_j (\mathbf{u}_j-\mathbf{u}_j).W(|x_j-x_i|)\, V_j$$

Scheme which is centered in space + fully explicit => need for stabilization

4. Stability? Pressure field quality?

centered + explicit => need for stabilization

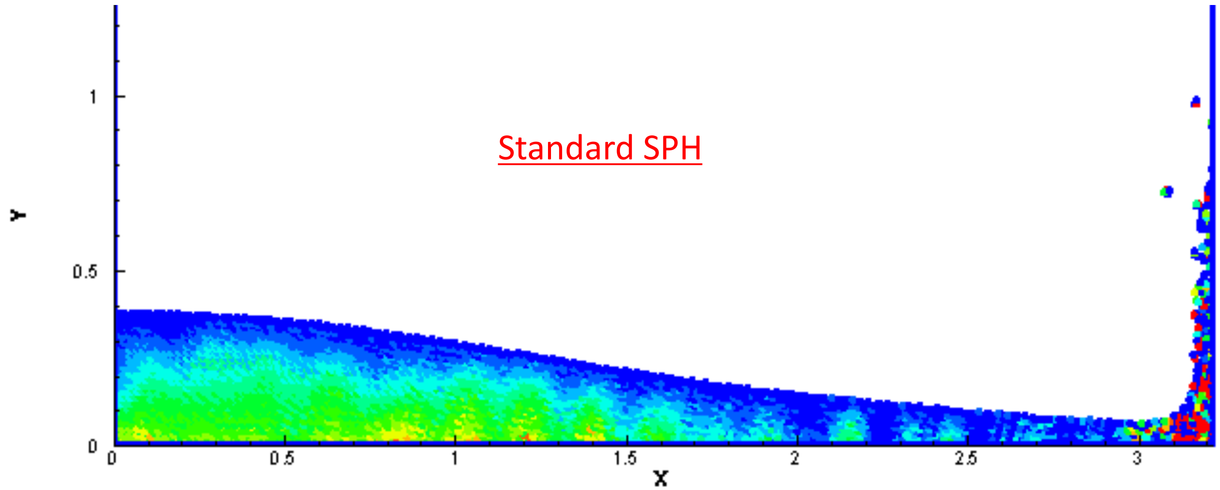
Method 1 (stable but not enough!)

last equations + artificial viscosity added through the pressure term (original Monaghan SPH scheme)

$$\rho_i \frac{d\vec{v}_i}{dt} = \rho_i \, \vec{g}_i - \sum_{j \in \Omega} (P_i + P_j) \, \vec{\nabla}_i \, W_{ij} \omega_j + \alpha h c_0 \rho_0 \sum_{j \in \Omega} \Pi_{ij} \, \vec{\nabla} \, W_{ij} \omega_j ,$$

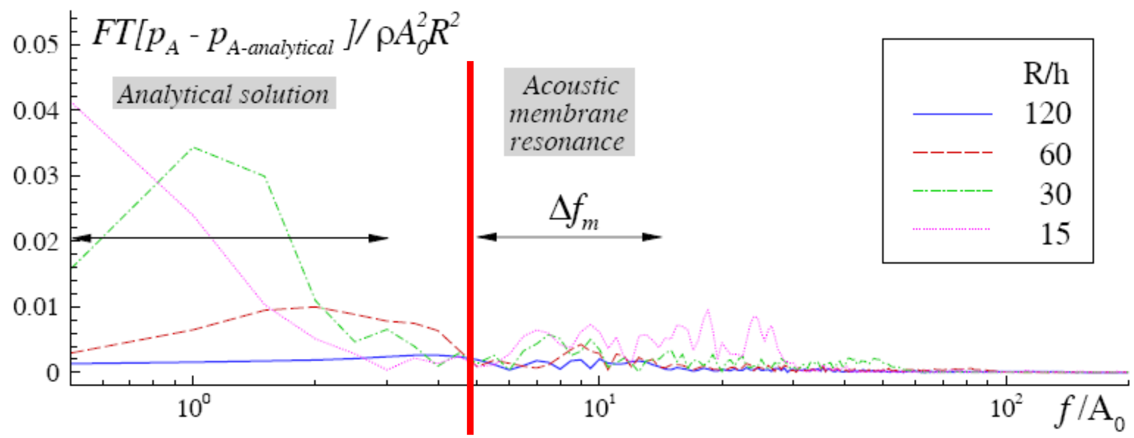
4. Stability? Pressure field quality?

Method 1 (artificial viscosity): Pressure field quality



Remember: we want pressure/force accuracy: we work for engineers, not gaming or movie people!

Where do the errors go? => acoustics



Le Touzé D. et al., A critical investigation of smoothed particle hydrodynamics applied to problems with free-surfaces, Int. J. Numer. Meth. Fluids **73**, 2013

4. Stability? Pressure field quality?

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Method 2

use of a density diffusive term (proportional to a Rusanov flux) in the continuity equation, e.g., δ-SPH, in addition to the artificial viscosity

$$\frac{d\rho_i}{dt} = -\rho_i \sum_{j\in\Omega} (\vec{v_j} - \vec{v_i}) . \vec{\nabla}_i W_{ij}\omega_j + \delta h c_0 \sum_{j\in\Omega} \vec{\psi_{ij}} . \vec{\nabla}_i W_{ij}\omega_j ,$$

Method 3

use of Riemann solvers (Vila, 1999) between each pair of particles (standard in FVM for hyperbolic systems)

$$\begin{cases} \left.\dfrac{d\overline{x}_i}{dt}\right|_{v_0} = \overline{v_{0i}} \\ \left.\dfrac{d\omega_i}{dt}\right|_{v^0} = \sum_{j\in P(\Omega)} \omega_i\omega_j \left(\overline{v_{0j}} - \overline{v_{0i}}\right).\nabla_i W_{ij} \\ \left.\dfrac{d\omega_i\overline{\phi}_i}{dt}\right|_{v^0} + \sum_{j\in P(\Omega)} \omega_i\omega_j \underbrace{\left(\overline{\overline{F}}_i - \overline{\phi}_i \otimes \overline{v_{0i}} + \overline{\overline{F}}_j - \overline{\phi}_j \otimes \overline{v_{0j}}\right)}_{\text{given by the Riemann problem solution}}.\nabla_i W_{ij} = \omega_i \overline{S}_i \end{cases}$$

Method 4

use an incompressible semi-implicit solution

Oger G. et al., SPH accuracy improvement through the combination of a quasi-Lagrangian shifting transport velocity and consistent ALE formalisms, J. Comput. Phys. **313**, 2016
Antuono et al., Free-surface flows solved by means of SPH schemes with numerical diffusive terms, Comput. Phys. Commun. **181**, 2010

4. Stability? Pressure field quality?

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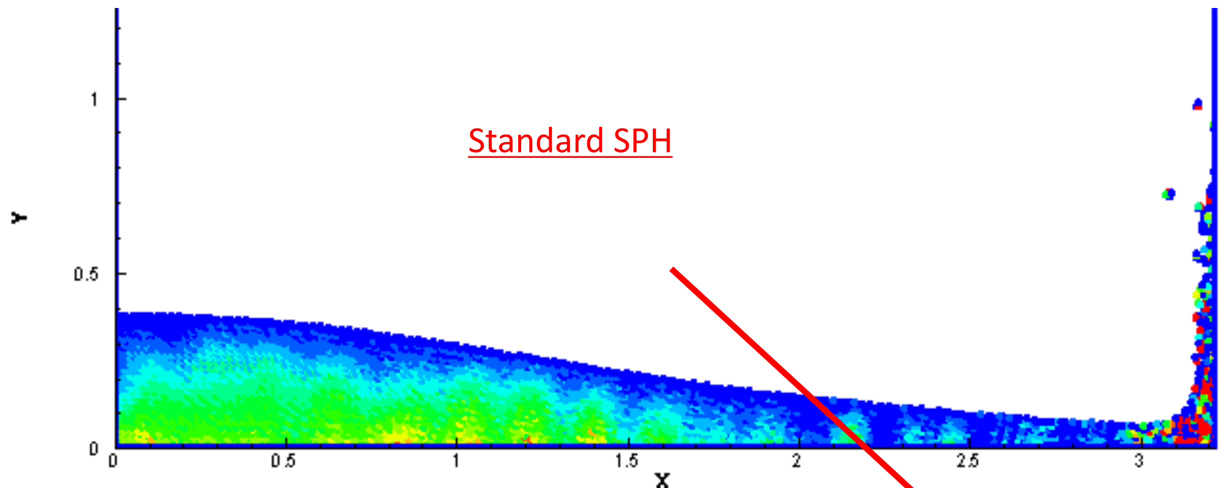
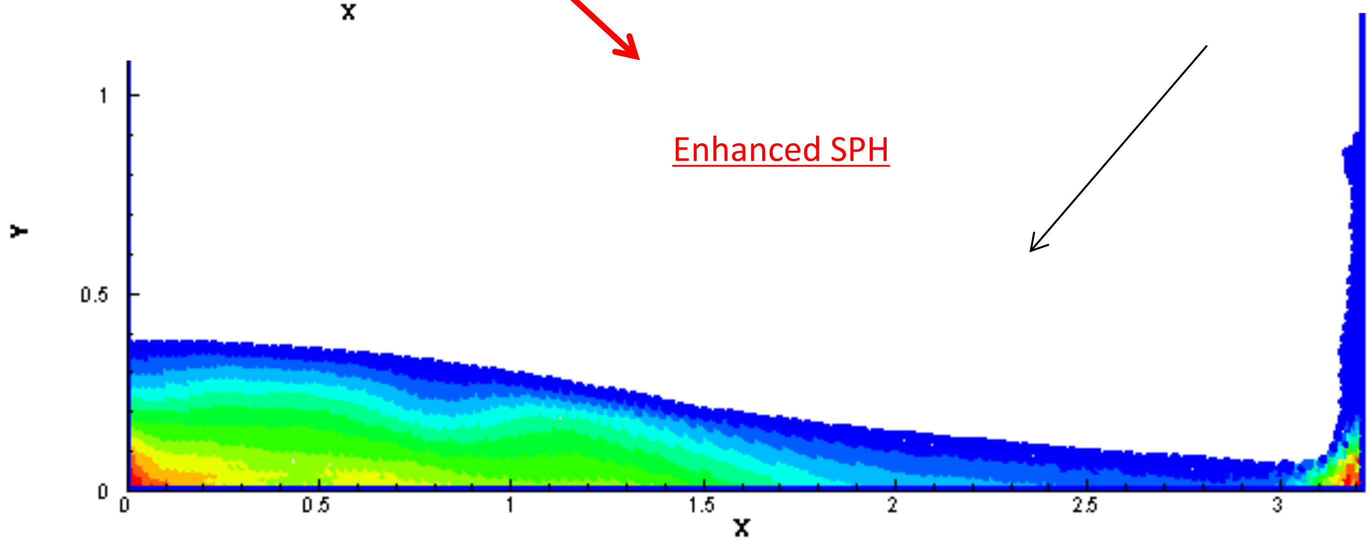
Methods 2 to 4 : Pressure field quality



Use of Riemann solvers

or density diffusive term,

or incompressible variant



SPH fact checking

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partly true	1.	SPH cannot be solved implicitly (as mesh-based) / it has very small time steps
partly true	2.	Weak-compressbility is an unphysical trick
not fully true	3.	SPH conserves everything
no more true	4.	SPH is unstable / SPH pressures are noisy
not so true	5.	SPH is not convergent
not true	6.	SPH is not accurate

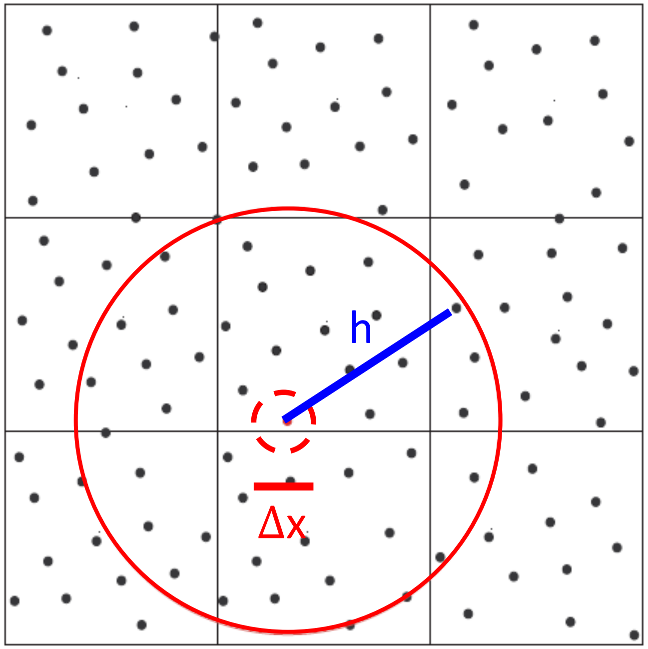
5. Convergence? 6. Accuracy?

A double convergence criteria

$$< u >_i \cong \int u(x) W(|x - x_i|)\, dx \cong \sum_j u_j W(|x_j - x_i|)\, dV_j$$

h -> 0

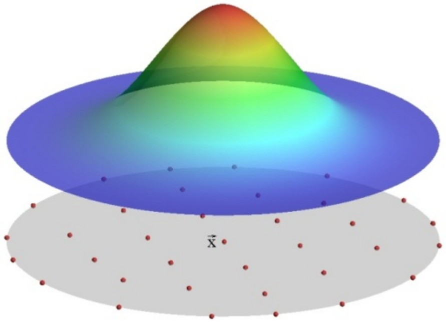
Δx/h -> 0



convergence order (Mas Gallic & Raviart):

$$h^2 + h^{-n} \left(\frac{\Delta x}{h} \right)^2$$

=> inconsistent if Δx/h = cst !!!
though common practice!

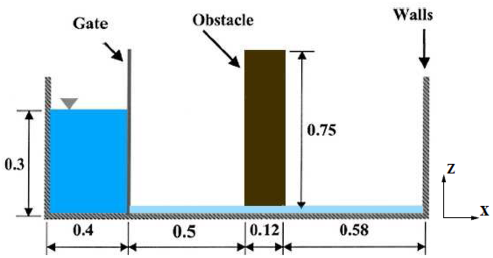


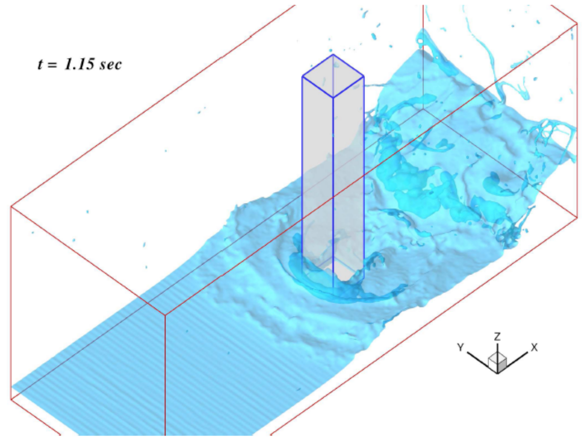
First-order operators (grad, div) used in standard SPH diverge at order 1 (pressure gradient) or are not convergent (velocity divergence)!!!

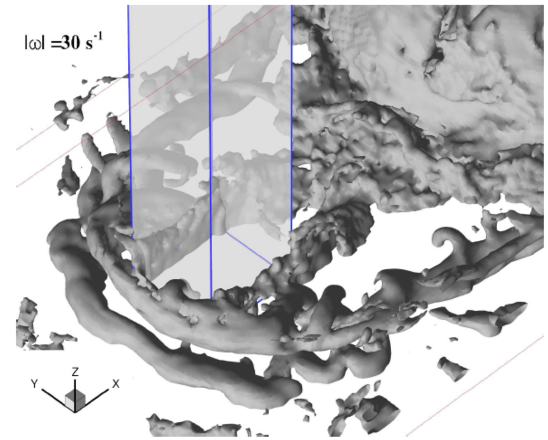
5. Convergence? 6. Accuracy?

And in practice, with $\Delta x/h$ = cst?

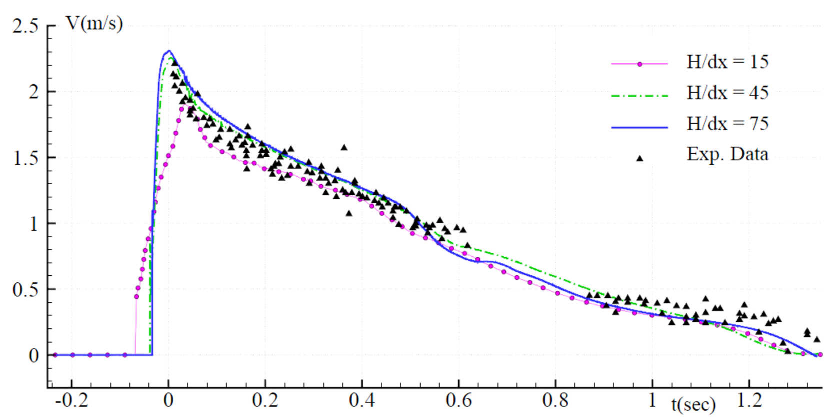
Validation on dam breaking test cases

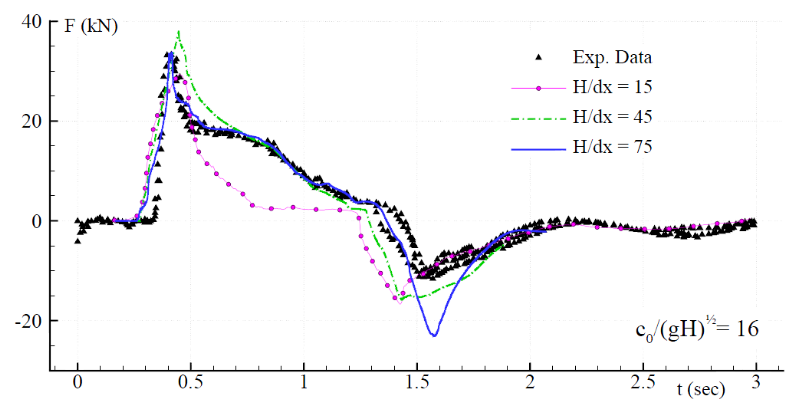






- Good prediction of velocity and forces:

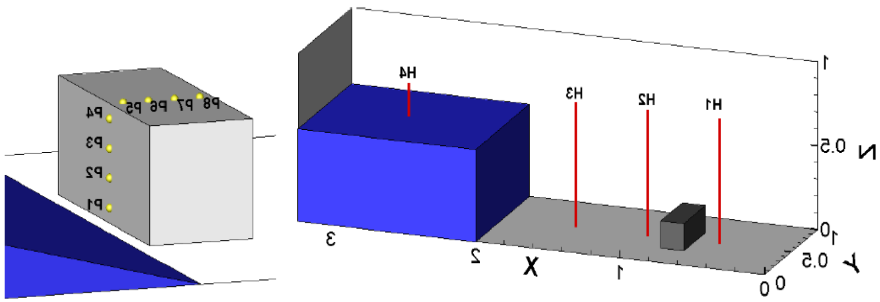


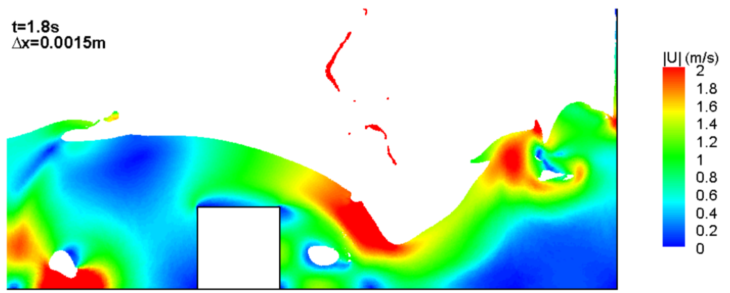


Marrone S. et al., δ-SPH model for simulating violent impact, Comput. Meth. Appl. Mech. Engng. **200**, 2011
Marrone S. et al., Fast free-surface detection and level-set function definition in SPH solvers, J. Comput. Phys. **229**, 2010

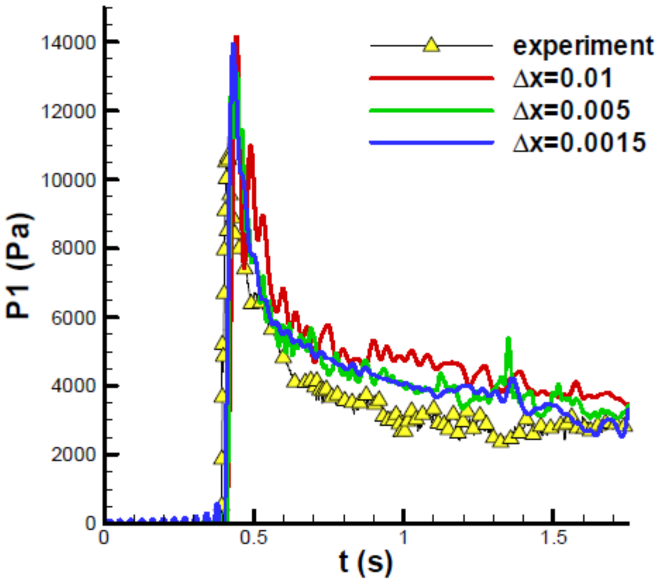
5. Convergence? 6. Accuracy?

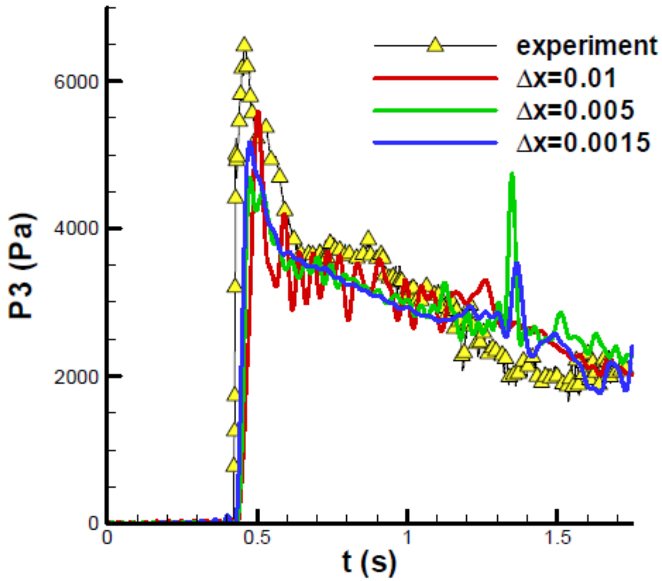
Validation on dam breaking test cases





- Good prediction of pressures as well:





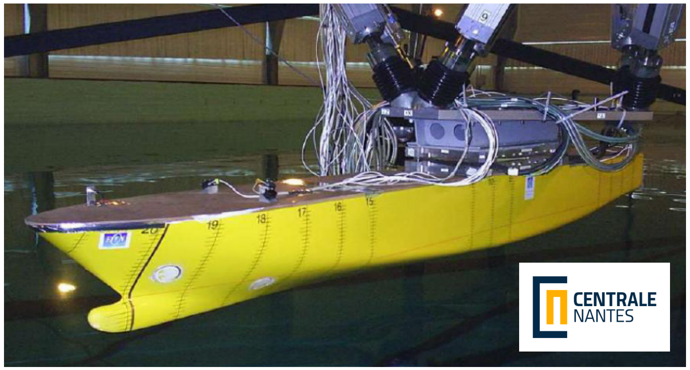
Heuristic convergence order is usually ~1

Marrone S. et al., δ-SPH model for simulating violent impact, Comput. Meth. Appl. Mech. Engng. **200**, 2011

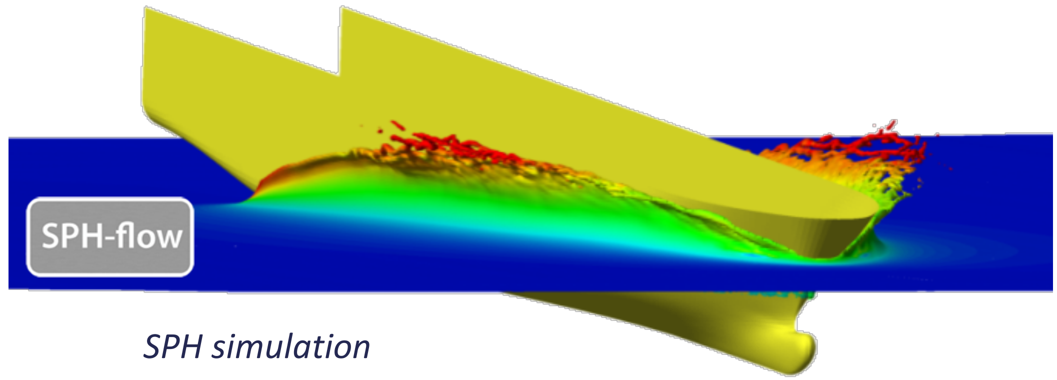
5. Convergence? 6. Accuracy?

LHEEA CENTRALE NANTES

Validation on slamming impact on a real application

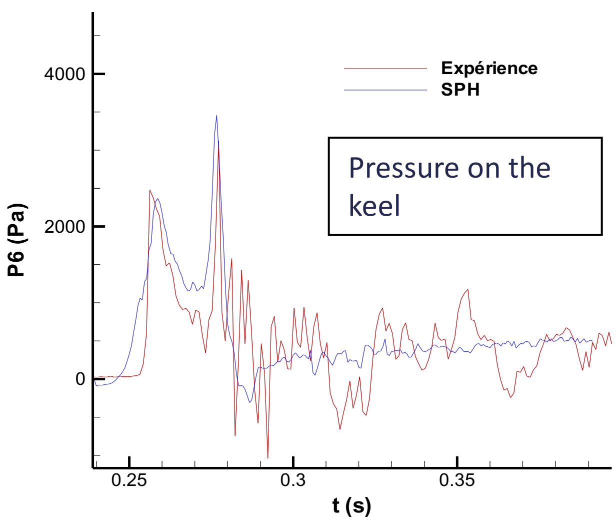


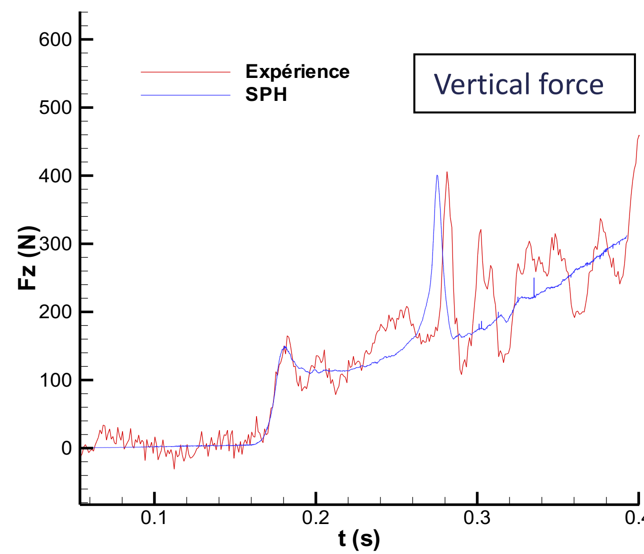
Experiment (ECN wave tank)



SPH simulation

- 3 million particles
- 6m/s impact (real scale)
- 250m ship (real scale)





Maruzewski et al., SPH high-performance computing simulations of rigid solids impacting the free-surface of water, J. Hydrau. Res. **48**, 2010

5. Convergence? 6. Accuracy?

So what can be the explanation?

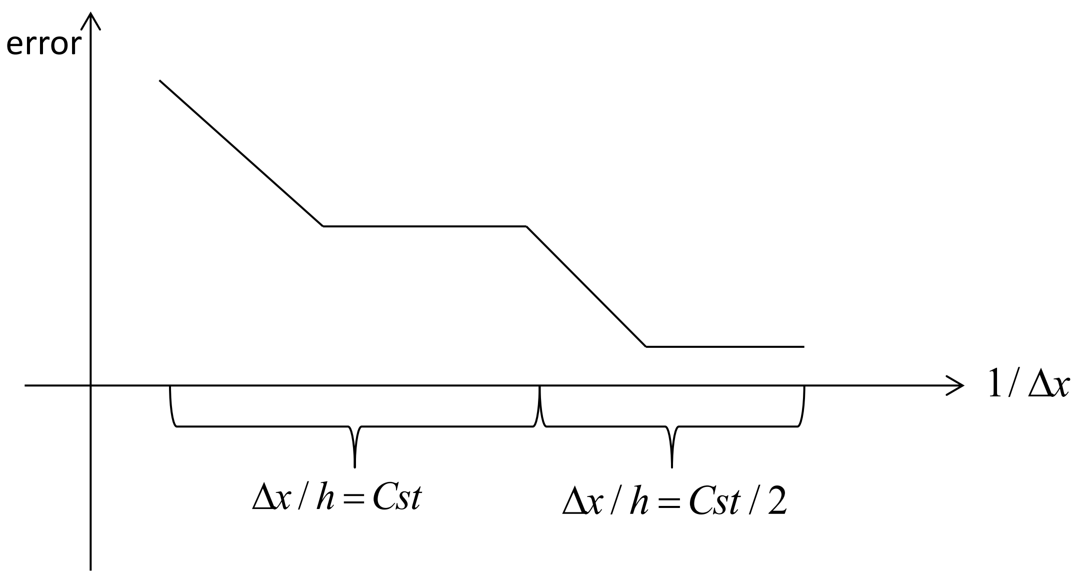
- Lagrangian! = no discretisation of the convection term $\frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{u}}{\partial t} + (\text{grad}\mathbf{u})\mathbf{u}$

  => exact convection

  => provides accuracy for flows dominated by convection (fast dynamics flows), e.g. 3D dambreak w/ 80k particles

  => convergence/accuracy is a mixed between exact (convection) and poor (pressure gradient, velocity divergence)

- Step-like convergence (Quinlan, Ellero... *et al.*) + already large stencil (250 neighbors in 3D!)



$\Rightarrow$ Heuristic convergence often of order 1 with « tolerable » saturation (problem dependent!)

$\Rightarrow$ + « saved » by conservation and exact convection

- Discretization preserving conservation of mass, momenta and total energy

SPH fact checking

partly true	1. SPH cannot be solved implicitly (as mesh-based) / it has very small time steps
partly true	2. Weak-compressbility is an unphysical trick
not fully true	3. SPH conserves everything
no more true	4. SPH is unstable / SPH pressures are noisy
not so true	5. SPH is not convergent
not true	6. SPH is not accurate
not yet!	7. SPH cannot be high order
false	8. SPH should be purely Lagrangian

7. Higher order? 8. Purely Lagrangian?

Operators can be easily corrected locally to increase their order of convergence (MLS…)

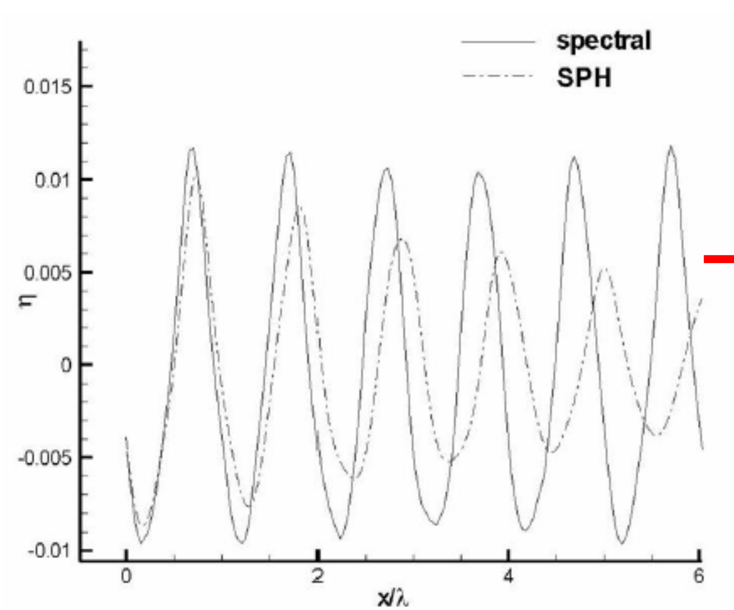
<u>principle:</u> imposing that the operator becomes exact for constant, linear, quadratic… fields

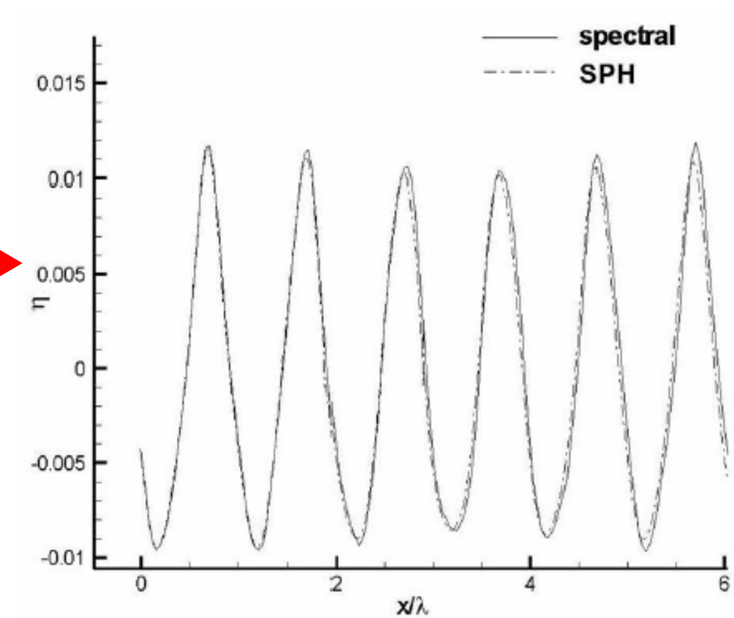
e.g.:

$$B_i = \left(\sum_{j \in \mathcal{P}} w_j (x_j - x_i) \otimes \nabla W_{ij} \right)^{-1}$$

$$\nabla W_{ij} \rightarrow \frac{1}{2} (B_i + B_j) \cdot \nabla W_{ij}$$

=> Efficiency on an academic example not meant to be solved by SPH: the propagation of a linear gravity wave (first-order convergent opeartors used)





7. Higher order? 8. Purely Lagrangian?

Consequences:

=> Convergence is theoretically recstored (see, e.g., Vila)

=> Computational cost is increased quite a lot (need to solve small matrices for each particle at each time step)

=> Typical corrections only restore order 1 (which was already heuristically obtained with reasonable saturation level) : order 2 is costly
=> more for accuracy than for convergence itself

=> Correction impacts other aspects, especially boundary conditions => not so easy

=> Calculations are often not improved/less stable!

7. Higher order? 8. Purely Lagrangian?

More accuracy = TOO Lagrangian

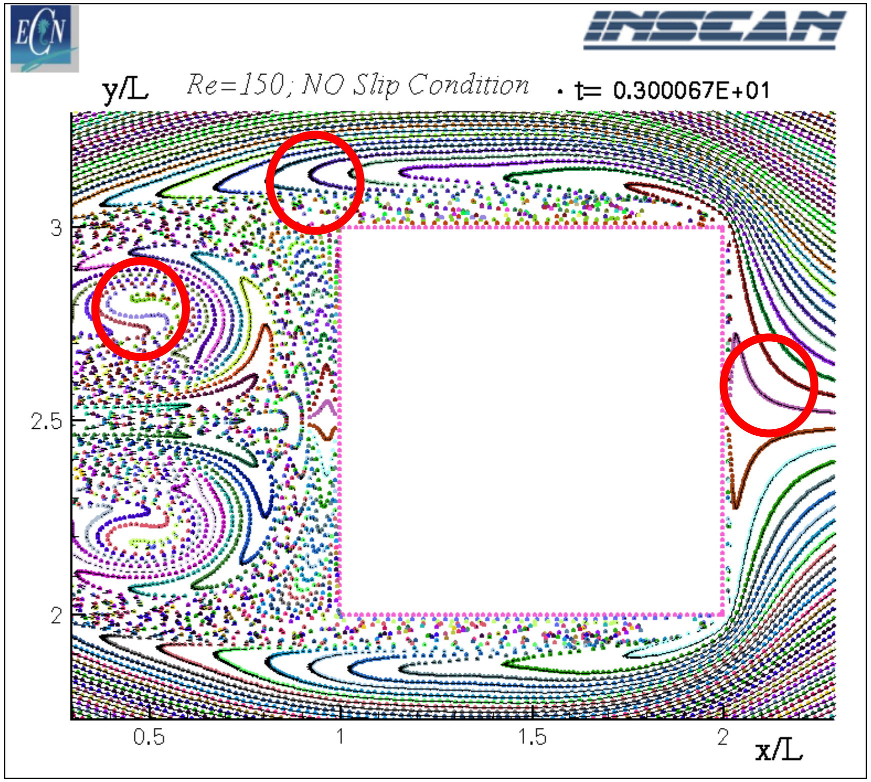
⇒ harmful effects on spatial interpolation, accuracy and stability

⇒ leads to numerical artefacts (especially on pressure fields)

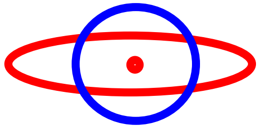
⇒ increases the numerical diffusion in the end

So, the better the worse! : the more accurate, the more Lagrangian and the more Lagrangian, the less accurate!

Together with difficulties at the boundaries, this explains why we do not see many high-order simulations...



- So why standard SPH particle distributions are so regular? => thanks to errors!
  - Hidden projection
  - Literature shows that errors on the pressure gradient induces a force tending to « fill voids »



7. Higher order? 8. Purely Lagrangian?

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Solution found in recent years when the schemes became more accurate

⇒ People often use arbitrary shifting (XSPH, shifting...) to homogeneize particle distribution in time
problem: it is not conservative and a lot comes from conservation!

⇒ We proposed a fully-conservative alternative through an ALE formulation (adapted from Vila, 1999 => Oger et al., J. Comput. Phys. **313**, 2016) used in a quasi-Lagrangian way (with same kind of displacements as shifting) => « consistent shifting »

$$\frac{d\vec{x}_i}{dt} = \vec{v}_{0i}, \tag{14}$$

$$\frac{d\omega_i}{dt} = \omega_i \sum_j (\vec{v}_{0j} - \vec{v}_{0i}) \nabla W_{ij}\, \omega_j, \tag{15}$$

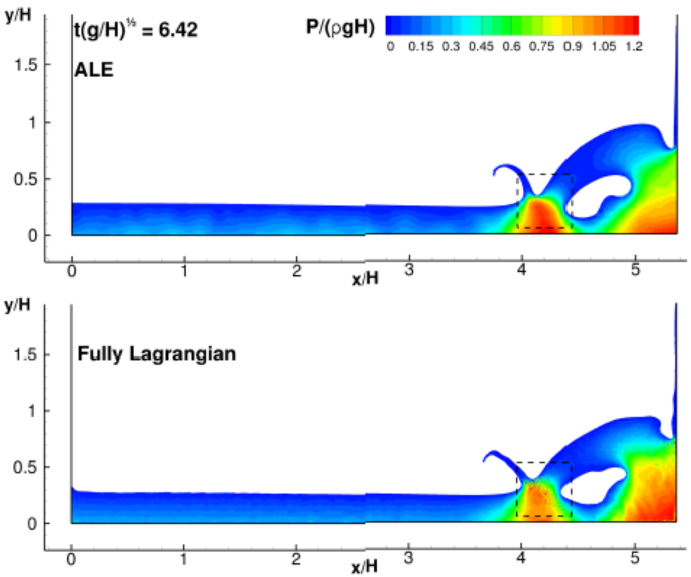
$$\frac{d\left(\omega_i \rho_i\right)}{dt} = -\omega_i \sum_j \left(\rho_i(\vec{v}_i - \vec{v}_{0i}) + \rho_j(\vec{v}_j - \vec{v}_{0j})\right) \nabla W_{ij}\, \omega_j, \tag{16}$$

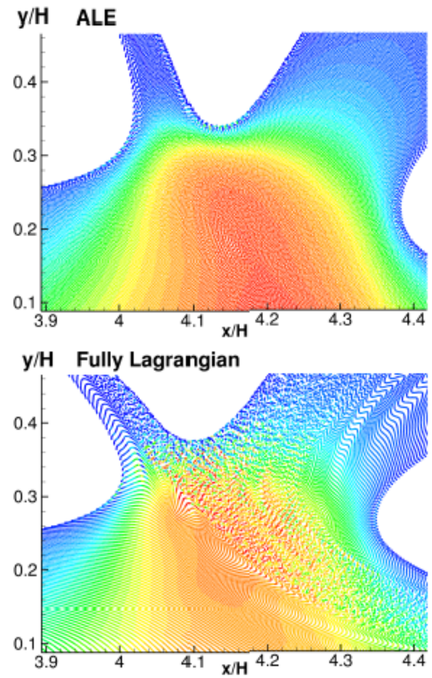
$$\frac{d\left(\omega_i \rho_i \vec{v}_i\right)}{dt} = -\omega_i \sum_j \left(\rho_i \vec{v}_i \otimes (\vec{v}_i - \vec{v}_{0i}) + \rho_j \vec{v}_j \otimes (\vec{v}_j - \vec{v}_{0j}) + P_i \bar{\bar{I}} + P_j \bar{\bar{I}}\right) \nabla W_{ij}\, \omega_j + \omega_i \rho_i \vec{g}. \tag{17}$$

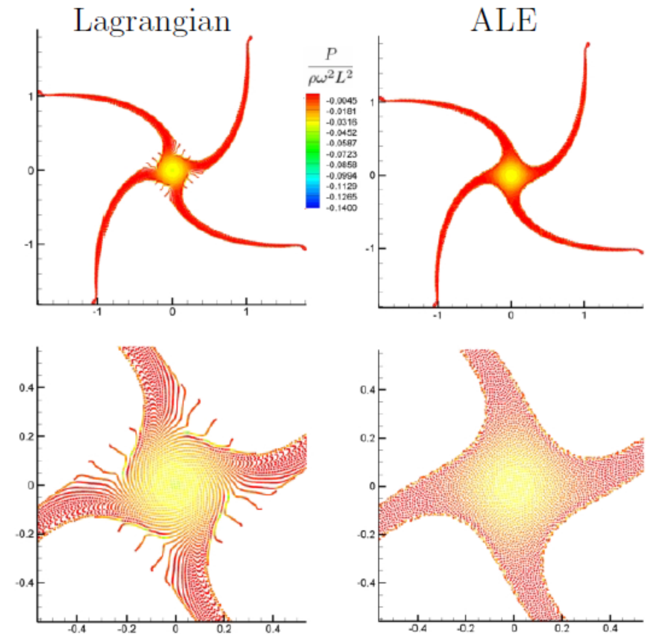
Oger G. et al., SPH accuracy improvement through the combination of a quasi-Lagrangian shifting transport velocity and consistent ALE formalisms, J. Comput. Phys. **313**, 2016

7. Higher order? 8. Purely Lagrangian?

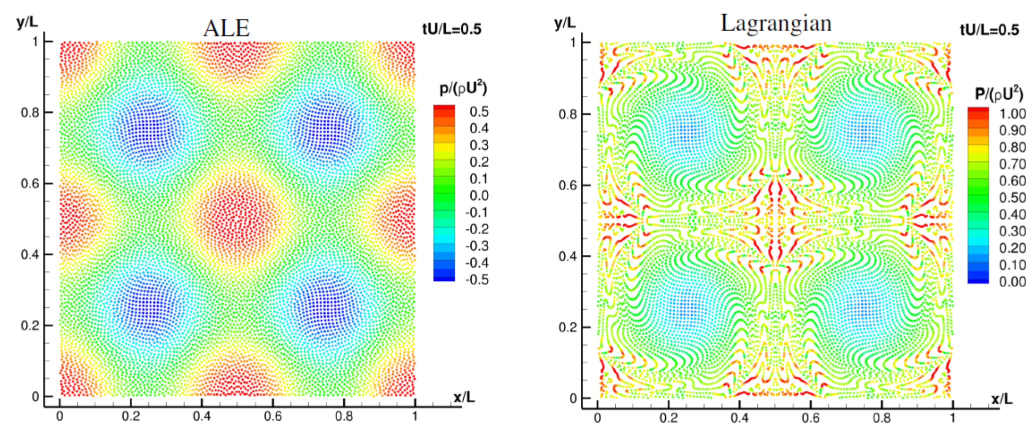
Examples of results with ALE formulation and Riemann solvers







With this ALE formulation we have all the ingredients to build a higher-order model (work in progress)



Oger G. et al., SPH accuracy improvement through the combination of a quasi-Lagrangian shifting transport velocity and consistent ALE formalisms, J. Comput. Phys. **313**, 2016

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not true	6. SPH is not accurate
not yet!	7. SPH cannot be high order
false	8. SPH should be purely Lagrangian
mainly false	9. Free-surface conditions are not modelled whereas they should

9.-10. Boundary conditions

=> mollification + discretisation

Incomplete support

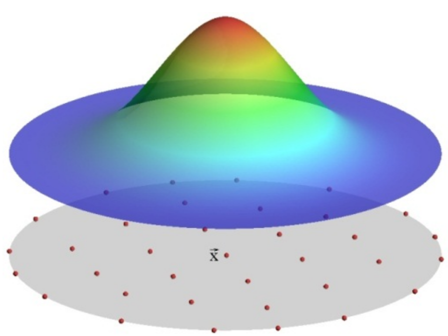
(1) (2)

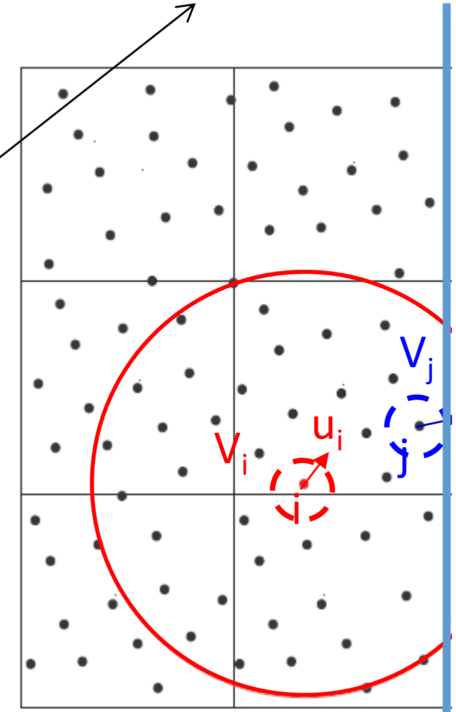
$$< \mathbf{u} >_i = \int \mathbf{u}(x)\delta(|x - x_i|)\, dx \cong \int \mathbf{u}(x)W(|x - x_i|)\, dx \cong \sum_j \mathbf{u}_j W(|x_j - x_i|)\, V_j$$

Known at the neighbour scattered points (particle)

Volume associated to the scattered points (particles)

Analytical function (kernel)





No more true

+ transfer of the differentiation to the kernel to get differential operators:

analytical

(1)

$$\nabla f(\vec{r}) \approx \int_D \nabla f(\vec{x}) W(\vec{r} - \vec{x}) d\vec{x} = \int_D f(\vec{x}) \nabla W(\vec{r} - \vec{x}) d\vec{x}$$

9.-10. Boundary conditions

=> boundary terms need to be accounted for in integration by parts

+

the support is no more filled by neighbours close to the boundary => potential inaccuracies

9. Free-surface boundary conditions

Kinematic FSBC is straightforward since we have a Lagrangian formalism

=> ok at convergence even though no particle strictly lies on the free surface due to their volume

We have proved that dynamic FSBC is verified in an integral sense provided appropriate operators are used for the pressure gradient, velocity divergence and velocity Laplacian (Colagrossi et al., Phys. Rev. E 2009 et 2011)

=> same reasoning as before but with boundary terms

for **Momentum** conservation (action/reaction)

$$< \nabla p >_i = \sum_j (p_j + p_i) W(|x_j - x_i|)\, V_j$$

$$< \mathrm{div}\mathbf{u} >_i = \sum_j (\mathbf{u}_j - \mathbf{u}_j) . W(|x_j - x_i|)\, V_j$$

for **Energy** conservation (Hamiltonian)

Colagrossi A. et al., Theoretical considerations on the free-surface role in the smoothed-particle-hydrodynamics model, Phys. Rev. E **79**, 2009

Colagrossi A. et al., Theoretical analysis and numerical verification of the consistency of viscous smoothed-particle-hydrodynamics formulations in simulating free-surface flows, Phys. Rev. E **84**, 2011

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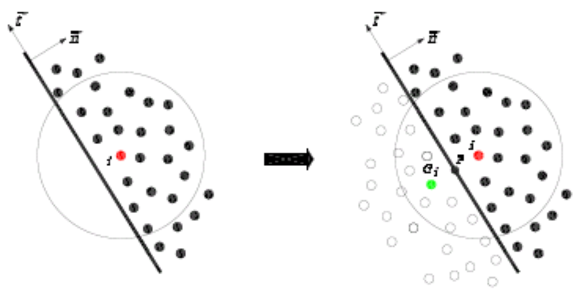
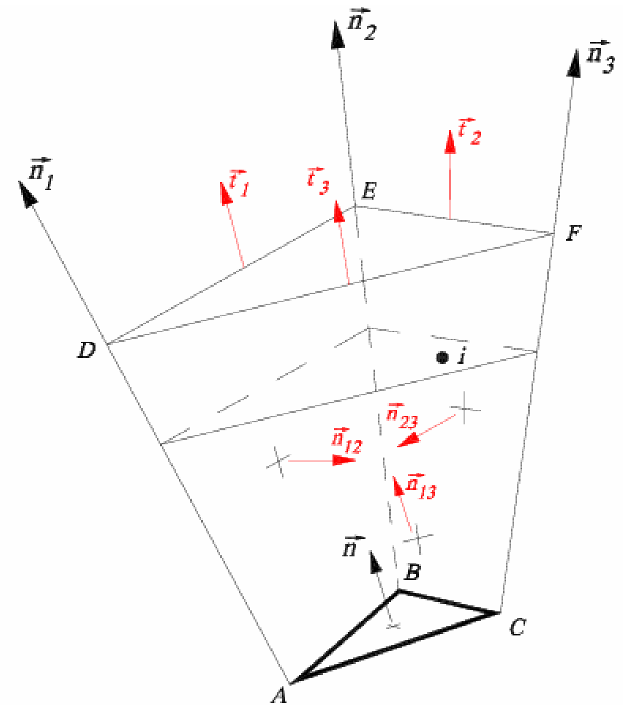
not yet!	7. SPH cannot be high order
false	8. SPH should be purely Lagrangian
mainly false	9. Free-surface conditions are not modelled whereas they should
no more true	10. There is no good scheme to model wall boundary conditions

10. Good wall boundary conditions?

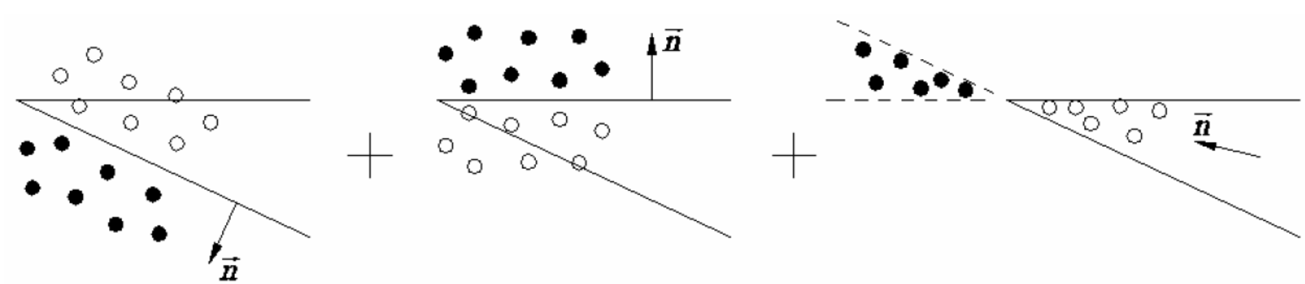
Formerly we used the ghost method, best compromise between generality and accuracy

=> 3D generic technique from any surface (e.g. IGES format)



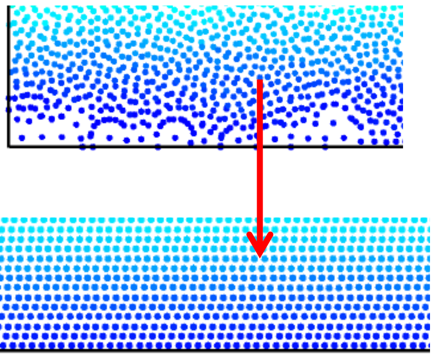


- Exact only for a flat panel, difficulties with geometrical singularities, especially sharp edges



10. Good wall boundary conditions?

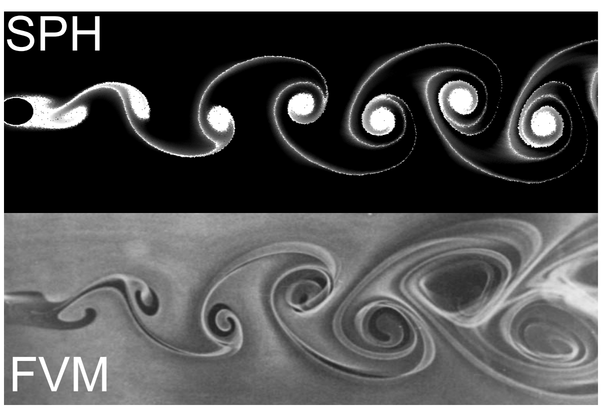
We proved that the no-slip ghost condition should not be applied to the velocity divergence to preserve the hyperbolicity of the system inviscid part (De Leffe et al., *6th SPHERIC workshop, 2011)*



$$\frac{dE}{dt} = -\sum_{i\in P(\Omega)} \sum_{j\in P(\Omega)\cup P(\partial\Omega)} m_i m_j \left(\frac{p_j \overline{v_i}}{\rho_j^{\,2}} + \frac{p_i \overline{v_j}}{\rho_i^{\,2}} + \frac{1}{2}\Pi_{ij}\left(\overline{v_i} + \overline{v_j}\right) \right) . \nabla_i W_{ij}$$

$$= \Delta E + \Delta E^{\Pi}$$

Validation on the flow past cylinder at Re=200



	Strouhal	C_d
SPH	2.0	1.47
Experiment	1.9	1.3

De Leffe M. et al., A modified no-slip condition in weakly-compressible SPH, Proc. 6th Int. SPHERIC Workshop, 2011

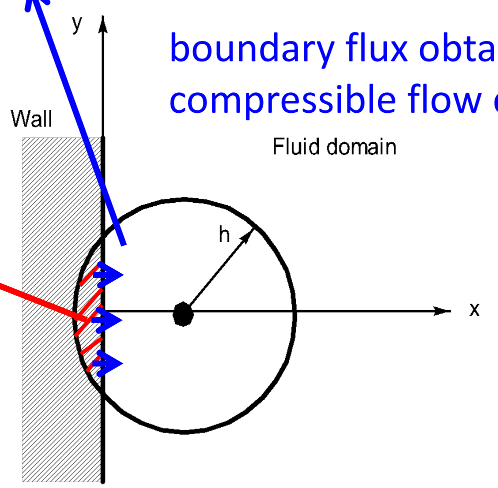
10. Good wall boundary conditions?

We proposed the Normal Flux Method (submitted to JCP)

$$
\begin{cases}
\left.\dfrac{d\overline{x_i}}{dt}\right|_{v_0} = \overline{v_{0i}} \\[2ex]
\left.\dfrac{d\omega_i}{dt}\right|_{v^0} = \dfrac{1}{\gamma_i}\displaystyle\sum_{j\in P(\Omega)} \omega_i\omega_j\left(\overline{v_{0j}}-\overline{v_{0i}}\right).\nabla_i W_{ij} + \dfrac{1}{\gamma_i}\displaystyle\sum_{j\in P(\partial\Omega)} \omega_i s_j\left(\overline{v_{0j}}-\overline{v_{0i}}\right).\overline{n_j}W_{ij} \\[2ex]
\left.\dfrac{d\omega_i\overline{\phi_i}}{dt}\right|_{v^0} + \dfrac{1}{\gamma_i}\displaystyle\sum_{j\in P(\Omega)} \omega_i\omega_j\left(\overline{\overline{F_i}}+\overline{\overline{F_j}}\right).\nabla_i W_{ij} + \dfrac{1}{\gamma_i}\displaystyle\sum_{j\in P(\partial\Omega)} \omega_i s_j\left(\overline{\overline{F_i}}+\overline{\overline{F_j}}\right).\overline{n_j}W_{ij} = \omega_i\overline{S_i}
\end{cases}
$$

compensates missing part of the kernel support

boundary flux obtained from compressible flow characteristics



- Belongs to the family of boundary integration techniques (like, e.g., USAW)
- Fully general technique
- No leakage of particles
- Permits very complex geometrical configurations meshed with millions of elements

Chiron L. et al., Accurate and efficient solid boundary conditions in SPH : the Normal Flux Method (NFM), submitted to J. Comput. Phys.

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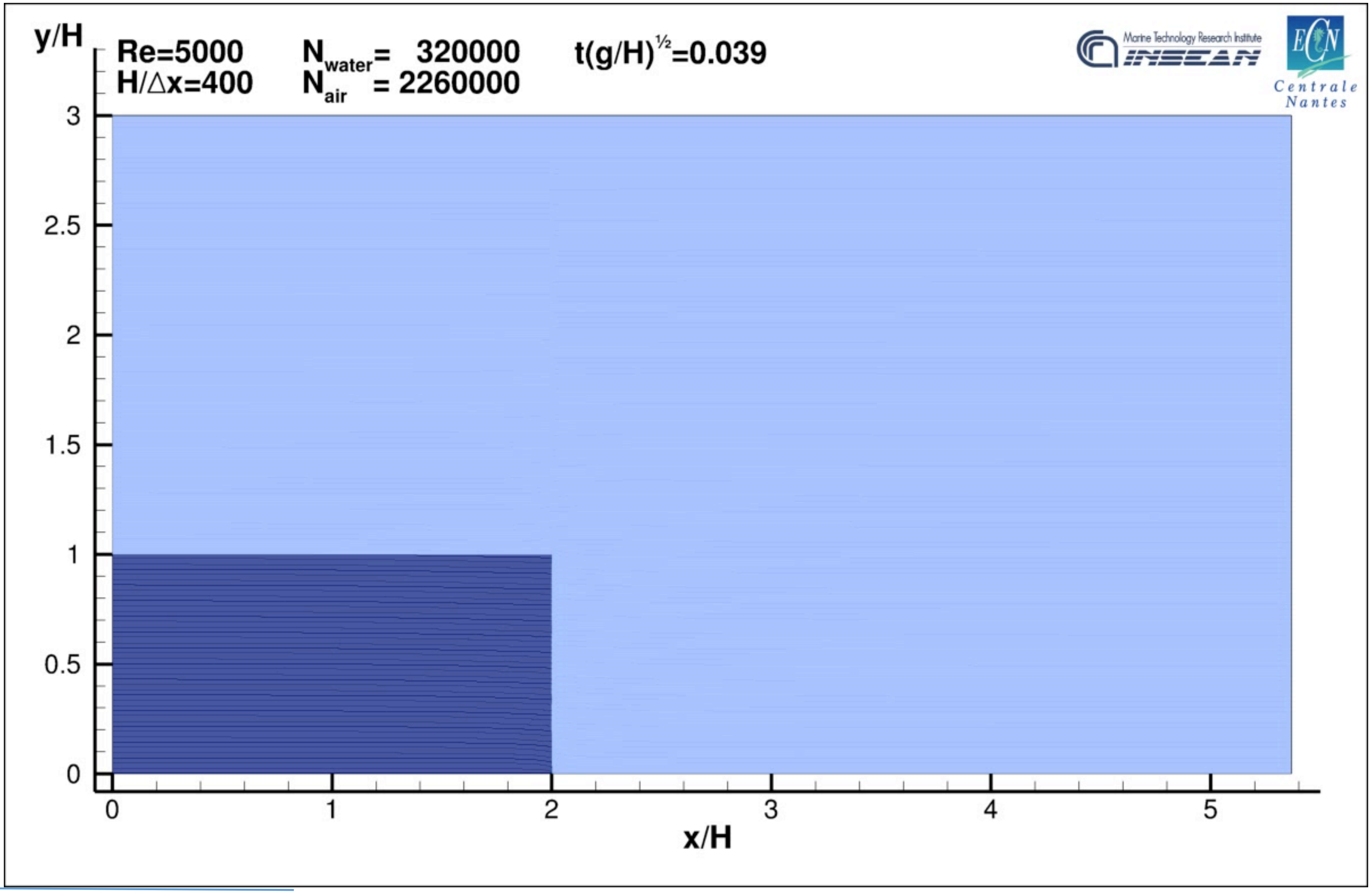
not so true	5. SPH is not convergent
not true	6. SPH is not accurate
not yet!	7. SPH cannot be high order
false	8. SPH should be purely Lagrangian
mainly false	9. Free-surface conditions are not modelled whereas they should
no more true	10. There is no good scheme to model wall boundary conditions
not so true	11. Single-phase assumption used in free-surface SPH is physically a non-sense

11. Validity of single phase approximation?

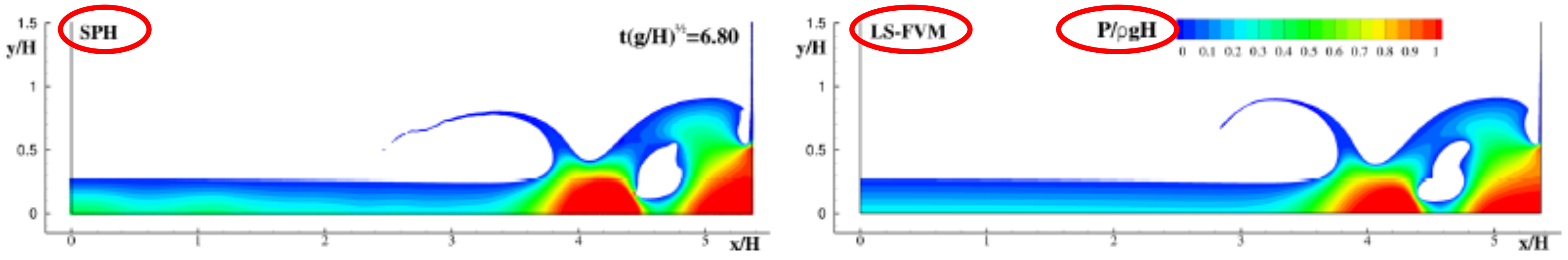
Lots of SPH users do not pay attention to the fact that single-phase simulation is a priori limited to non-breaking free-surface flows

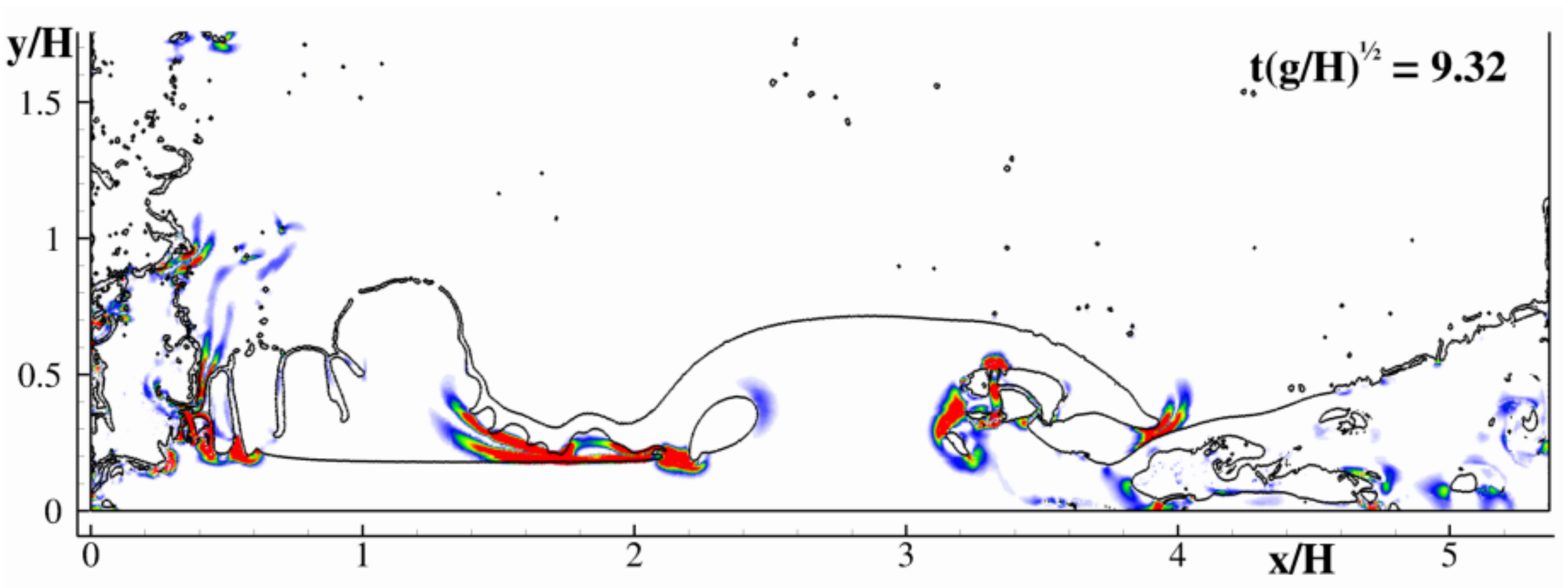
=> Study of a complex dambreak flow with 1 and 2 phases simulated



Marrone S. et al., Analysis of free-surface flows through energy considerations: Single-phase versus two-phase modeling, Phys. Rev. E **93**, 2016

11. Validity of single phase approximation?





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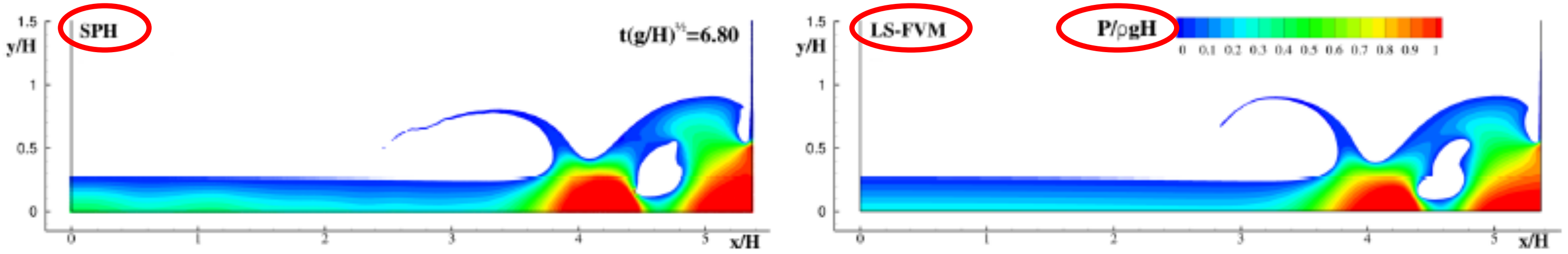

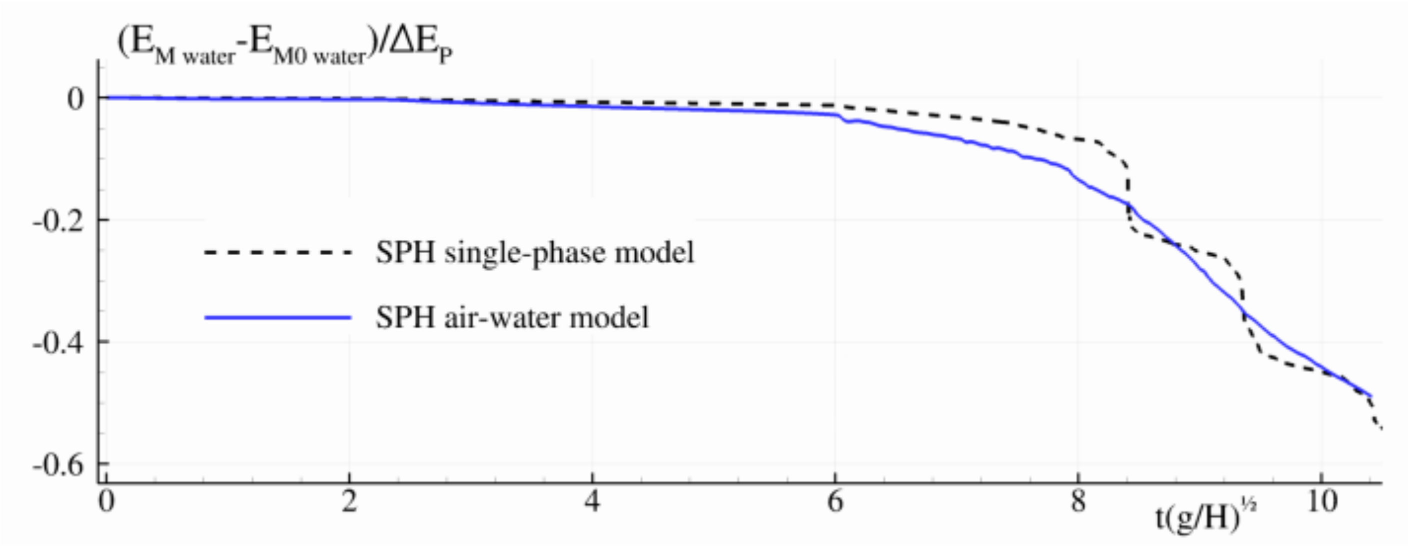

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false	8.	SPH should be purely Lagrangian
mainly false	9.	Free-surface conditions are not modelled whereas they should
no more true	10.	There is no good scheme to model wall boundary conditions
not so true	11.	Single-phase assumption used in free-surface SPH is physically a non-sense
still true	12.	SPH is costly / any future tracks?

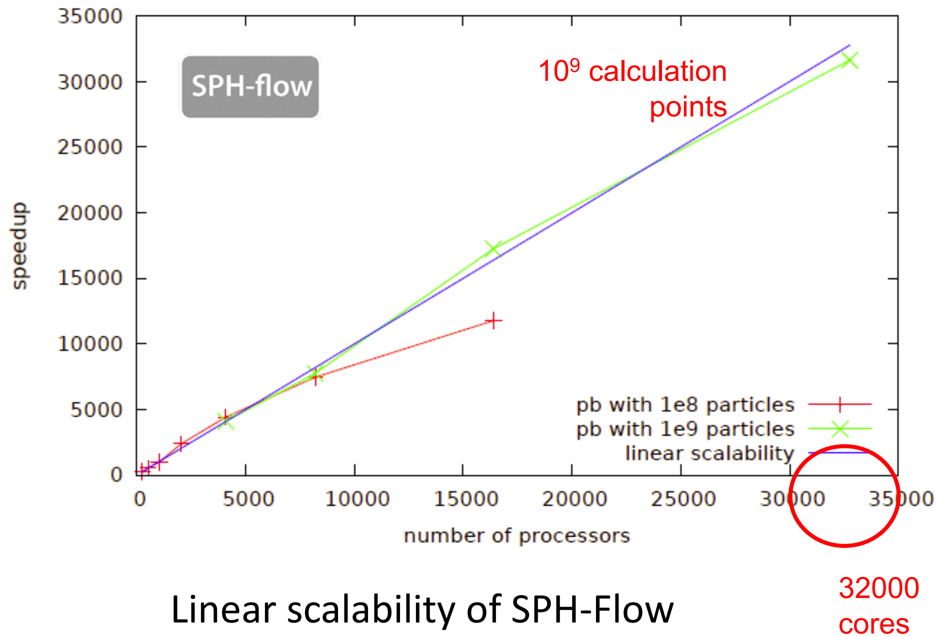
12. SPH is costly?

Unfortunately : yes!

- Very large stencils (typically 50 neighbours in 2D and 250 in 3D) => maybe less when we will have a robust second-order scheme?
- Small time steps => maybe a fully-implicit variant some day?

⇒ Compete well only where mesh-based methods have difficulties and for fast dynamic flows

⇒ Need for large hardware/efficient strategy, e.g. MPI/OpenMP or GPGPU

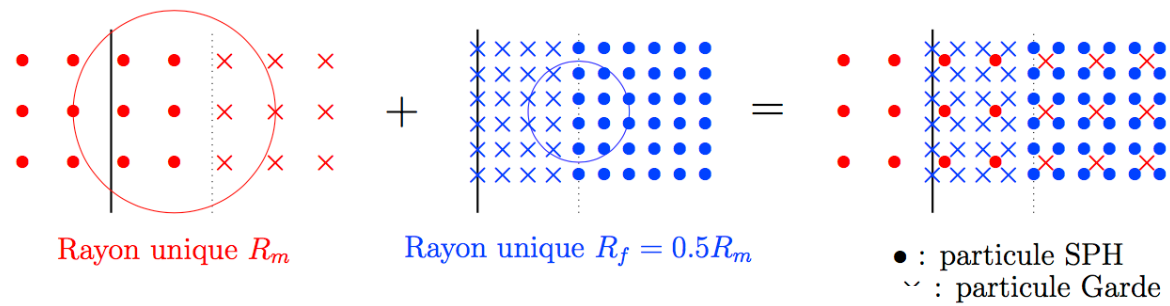


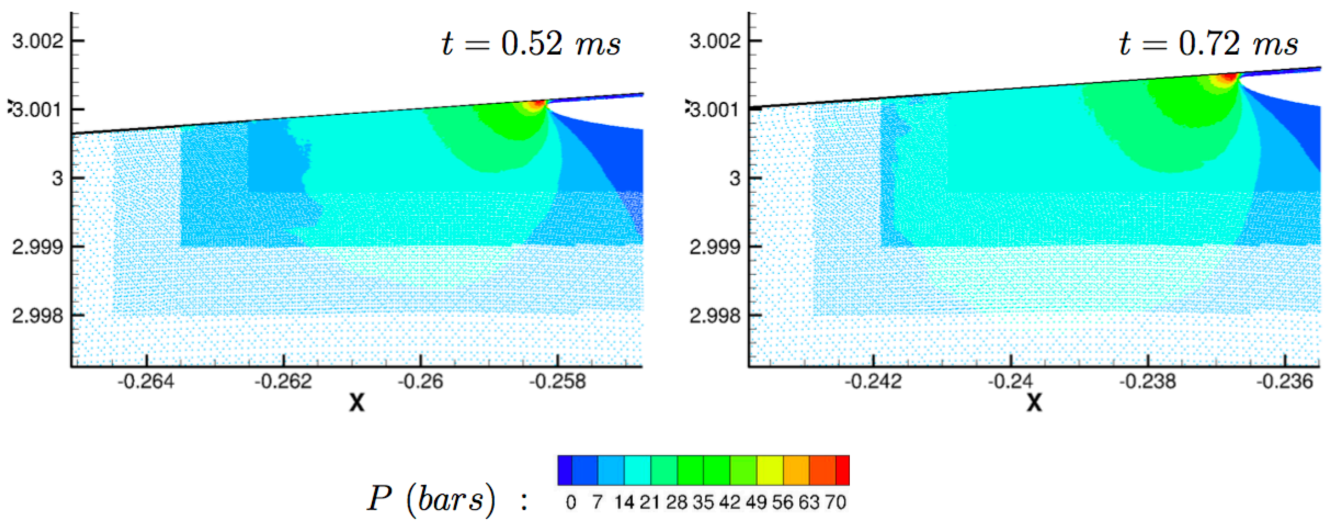
Linear scalability of SPH-Flow

12. Future tracks

Adaptive particle refinement (APR)

- Inspired from mesh-based AMR, but adapted to a Lagrangian formalism => use of « guard particles », prolongations, restrictions. as in AMR
- Has proved to be accurate, efficient and robust



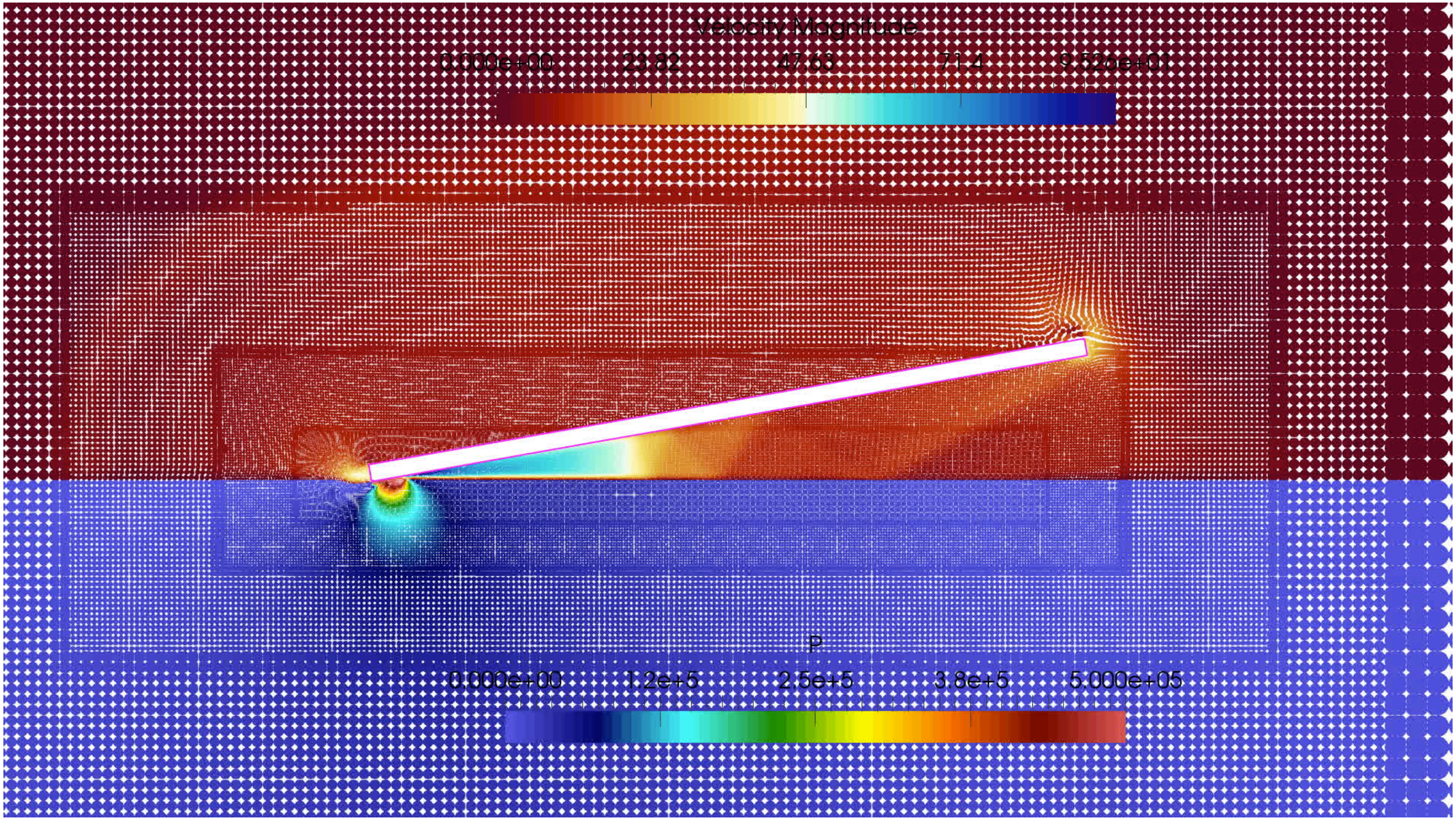


Barcarolo et al., Adaptive particle refinement and derefinement applied to the smoothed particle hydrodynamics method, J. Comput. Phys. **273**, 2014

Chiron et al., Analysis and improvements of Adaptive Particle Refinement (APR) through CPU time, accuracy and robustness considerations, to appear in J. Comput. Phys. , 2017

12. Future tracks

Adaptive particle refinement (APR): 40 m/s plate ditching (two-phase model)



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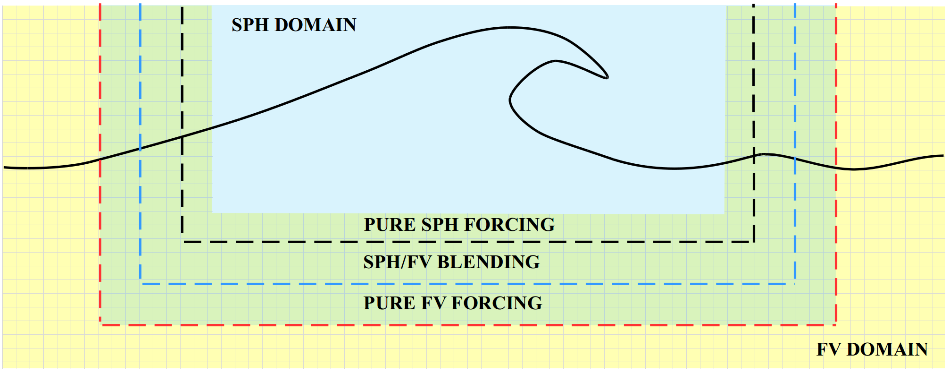
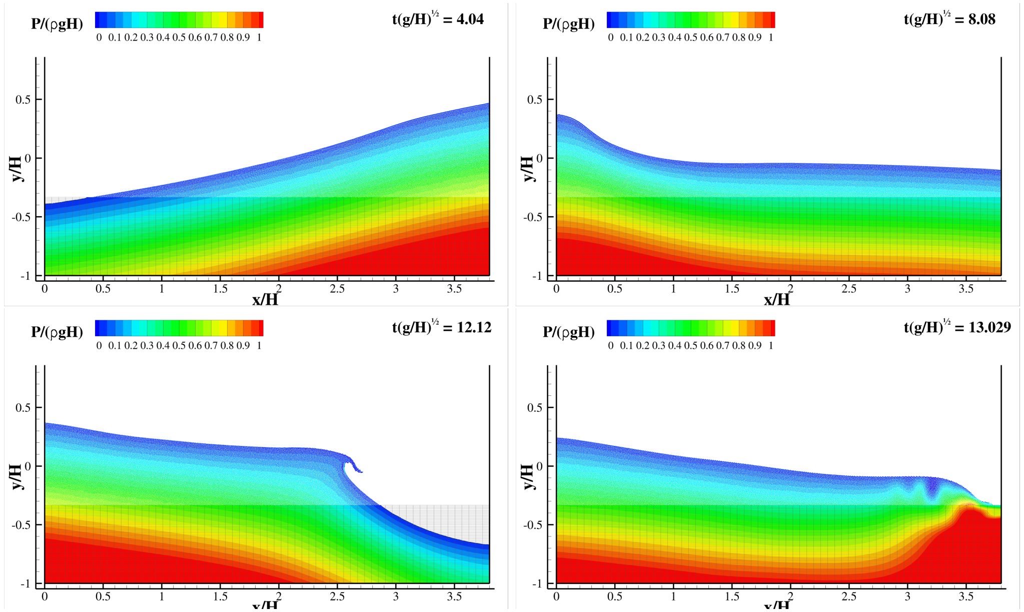
12. Future tracks

Coupling to Finite Volumes

- Coupling with a finite volume level-set solver
- Principle : use of forcing and blending zones
- Efficient even when the change of solver intersects the free surface
- Validated on numerous 2D test cases





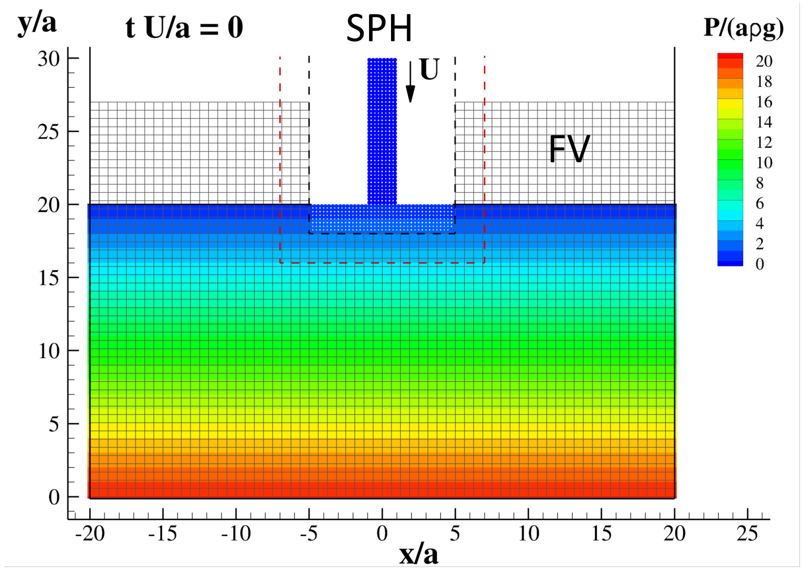
Marrone S. et al., Coupling of Smoothed Particle Hydrodynamics with Finite Volume method for free-surface flows, J. Comput. Phys. **310**, 2016

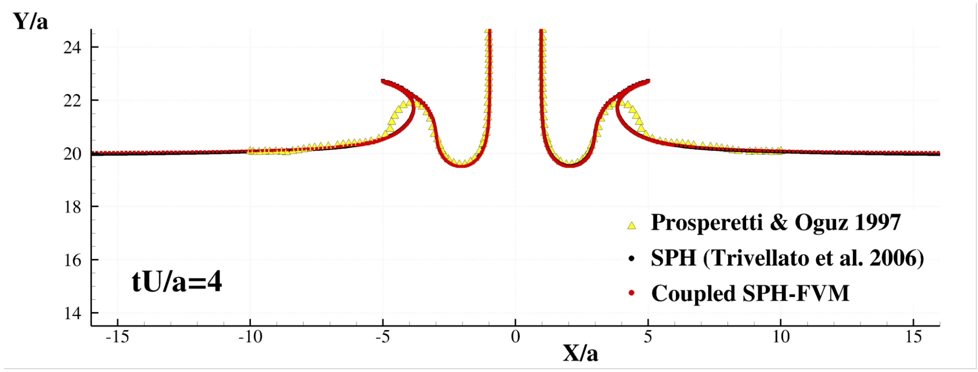
Marrone S. et al., Coupled SPH-FV method with net vorticity and mass transfer, submitted to J. Comput. Phys.

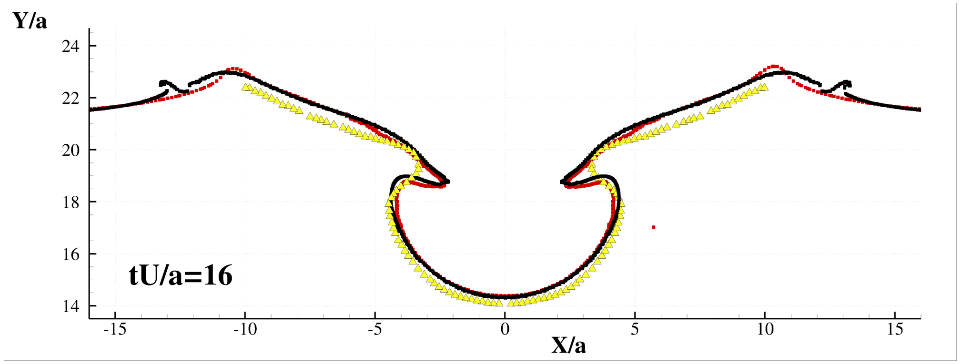
12. Future tracks

Coupling to Finite Volumes

Free-surface (Froude) driven test-case (difficult for the FV level-set solver)







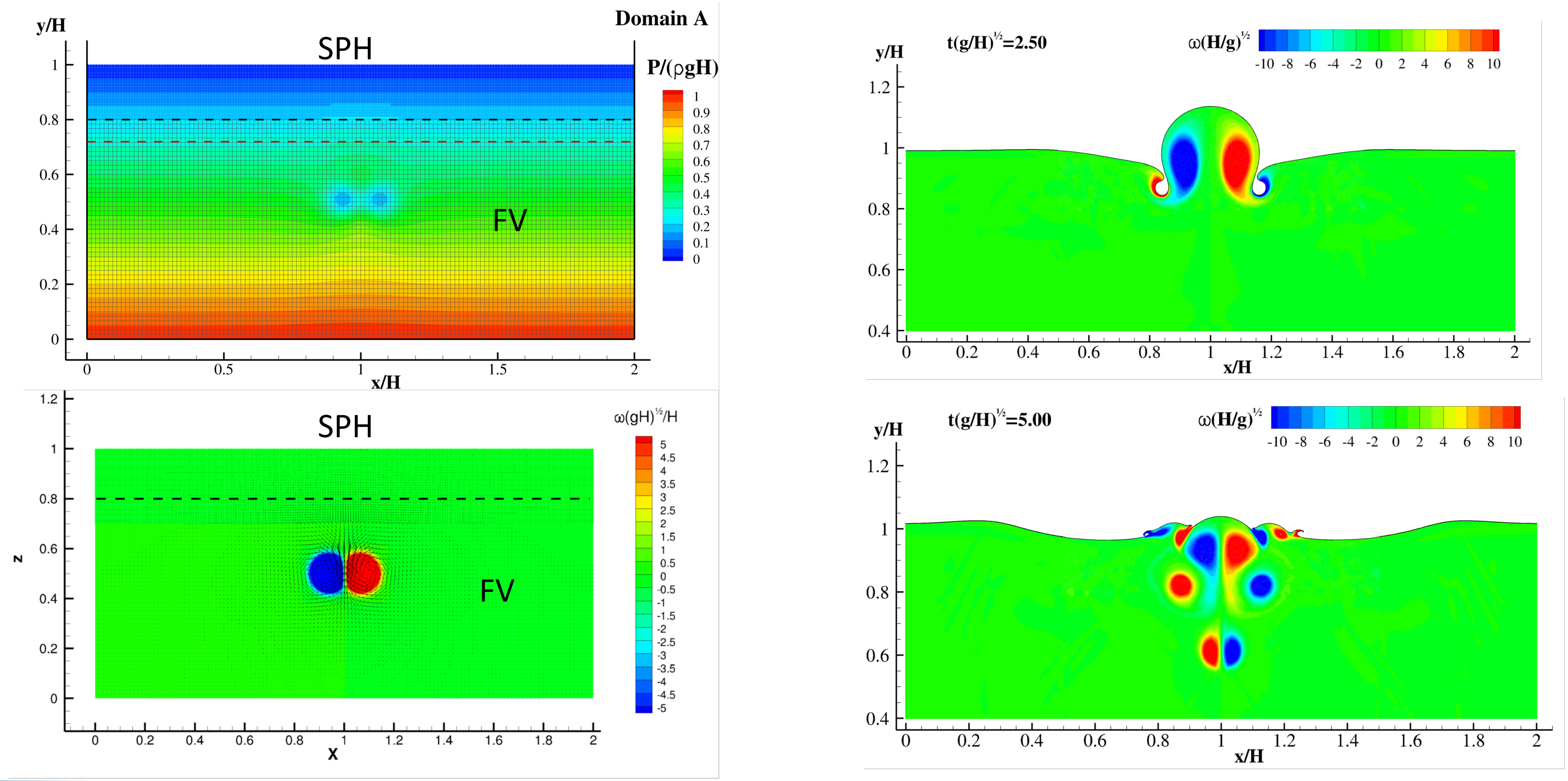
Marrone S. et al., Coupling of Smoothed Particle Hydrodynamics with Finite Volume method for free-surface flows, J. Comput. Phys. **310**, 2016

Marrone S. et al., Coupled SPH-FV method with net vorticity and mass transfer, submitted to J. Comput. Phys.

12. Future tracks

Coupling to Finite Volumes

Vorticity-driven (Reynolds) test-case (difficult for the SPH solver)



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SPH fact checking

partly true	1. SPH cannot be solved implicitly (as mesh-based) / it has very small time steps
partly true	2. Weak-compressbility is an unphysical trick
not fully true	3. SPH conserves everything
no more true	4. SPH is unstable / SPH pressures are noisy
not so true	5. SPH is not convergent
not true	6. SPH is not accurate
not yet!	7. SPH cannot be high order
false	8. SPH should be purely Lagrangian
mainly false	9. Free-surface conditions are not modelled whereas they should
no more true	10. There is no good scheme to model wall boundary conditions
not so true	11. Single-phase assumption used in free-surface SPH is physically a non-sense
still true	12. SPH is costly / any future tracks?
no more true	13. SPH is a research object which has no industrial potential

13. Industrial applications?

What is true

- SPH is costly and not accurate for all problems
  => it has a restricted field of applications (for now and probably for a long time)

- The fields of applications are:
  – Fast dynamics problems: small time steps + exact convection + complex interfaces
  – Complex physics: multi-body in the flow with contact, problems with different species/physical phenomena (explicit solving)
  – Multi-solver problems: easy coupling with other solvers: SPH-FEM / SPH-DEM / …

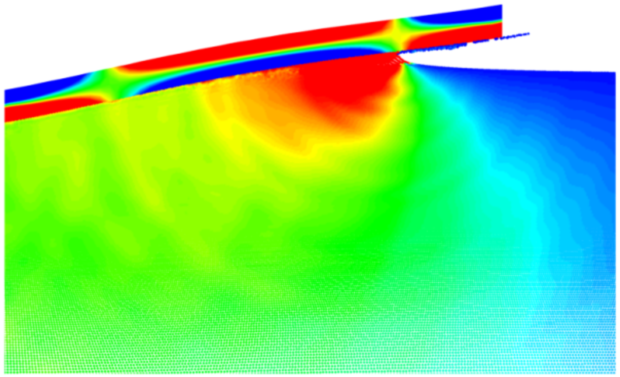
- Another asset: CAO to CFD (like LBM)

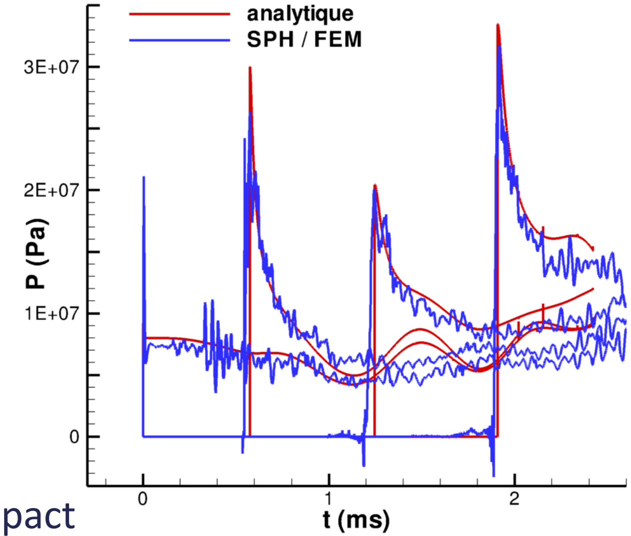
A rather extensive review: Shadloo et al., Computers & Fluids **136**, 2016

Shadloo M.S., Smoothed particle hydrodynamics method for fluid flows, towards industrial applications: Motivations, current state, and challenges, Computers & Fluids **136**, 2016

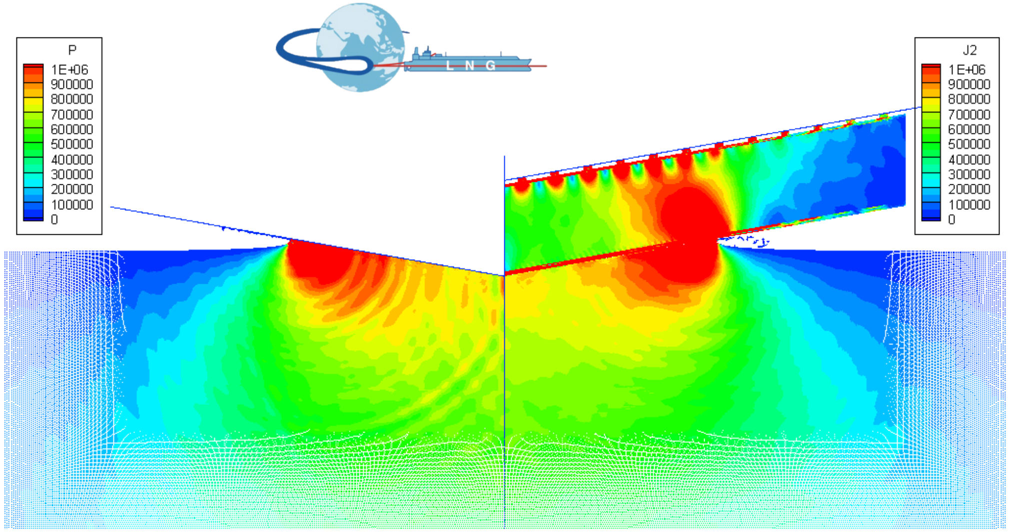
13. Industrial applications?

Fast fluid-structure coupled impact





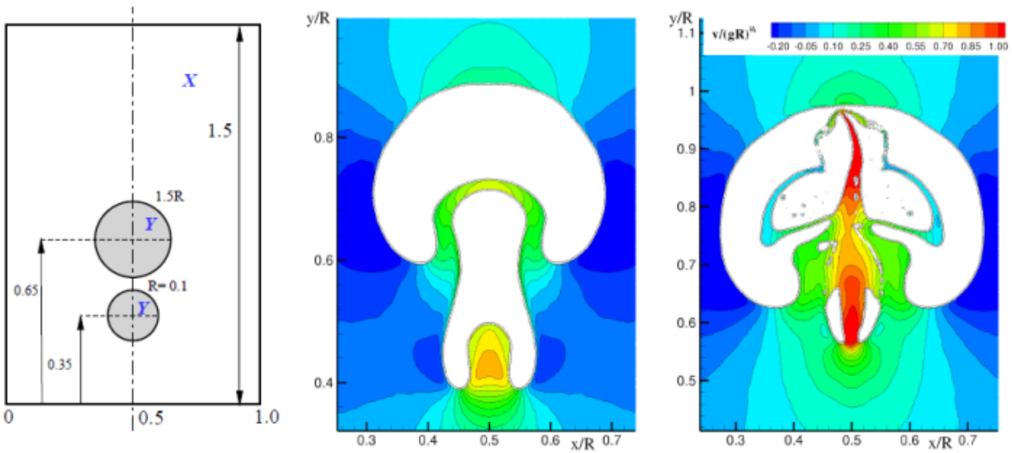
Fast aluminium beam impact

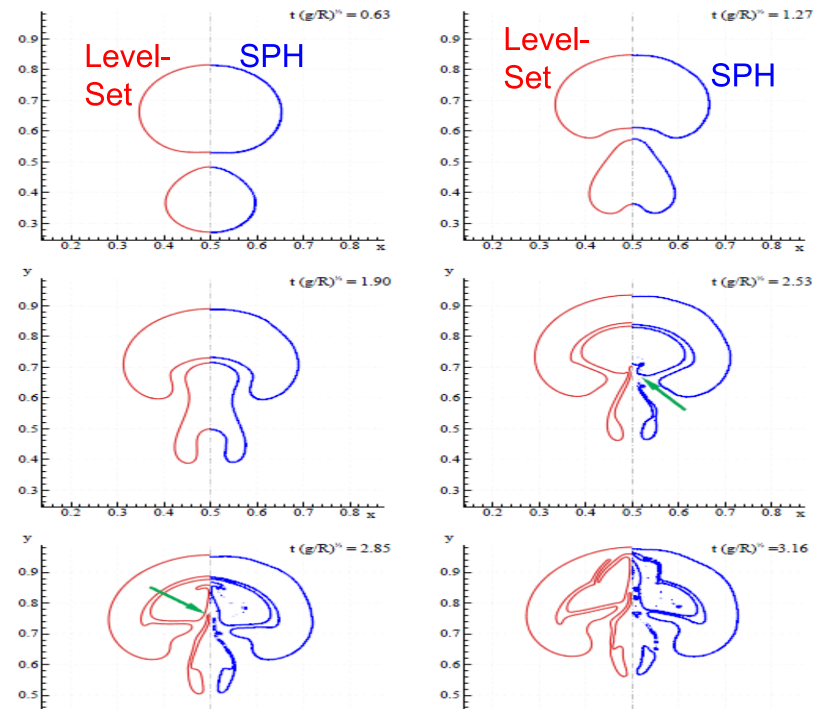


LNG membrane impact

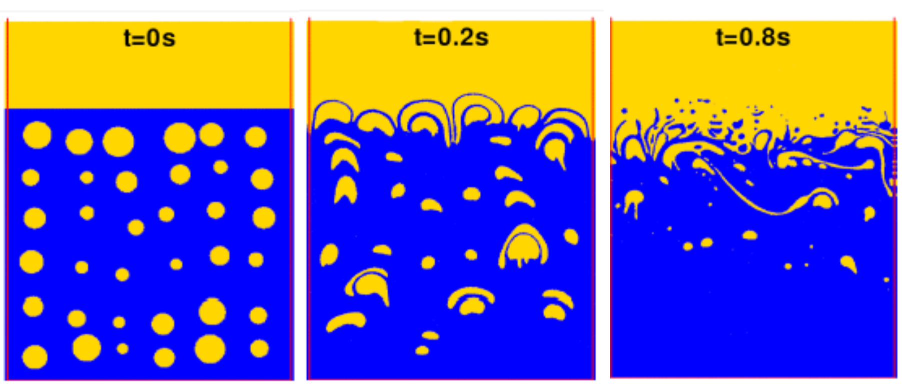
Exemples d'application

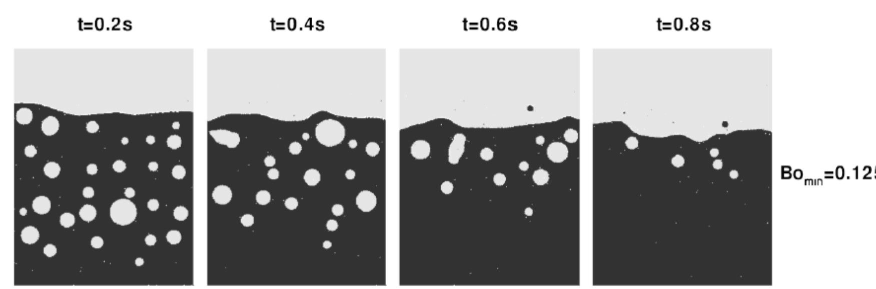
Complex bubbly flows (separation, atomisation)





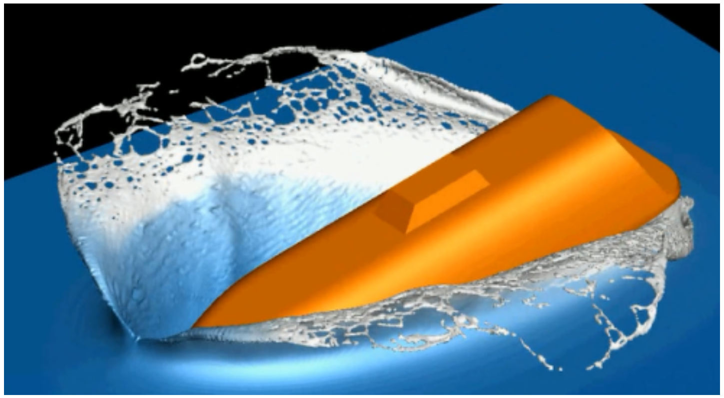
Water-oil separation (w/o or w/ surface tension)



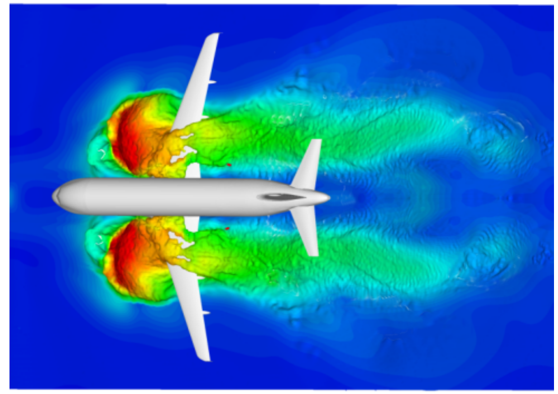


Grenier et al., Viscous bubbly flows simulation with an interface SPH model, Ocean Engng. **69,** 2013

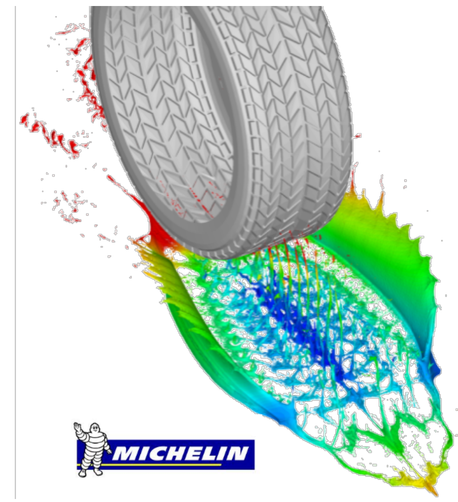
13. Industrial applications?



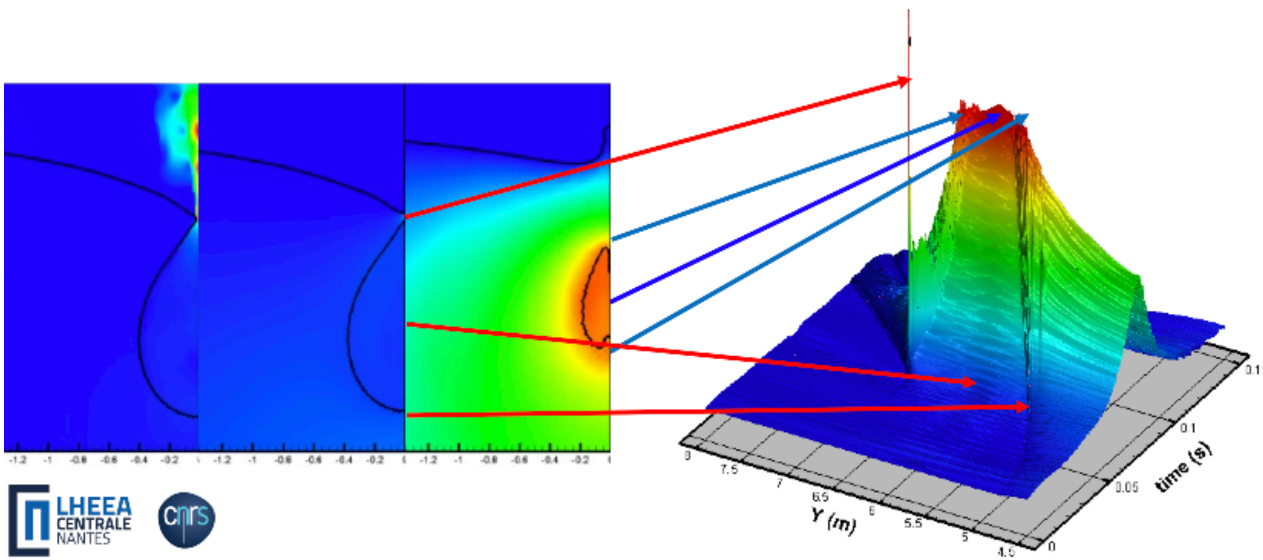
Lifeboat launching

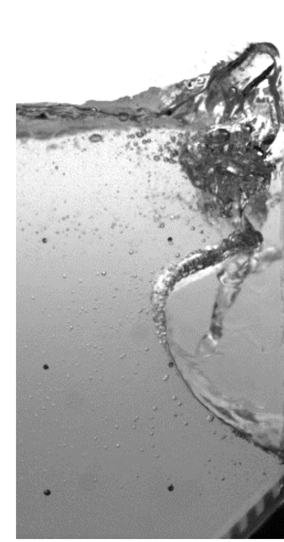


Aircraft ditching



Hydroplaning



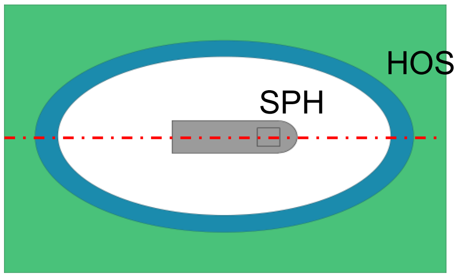


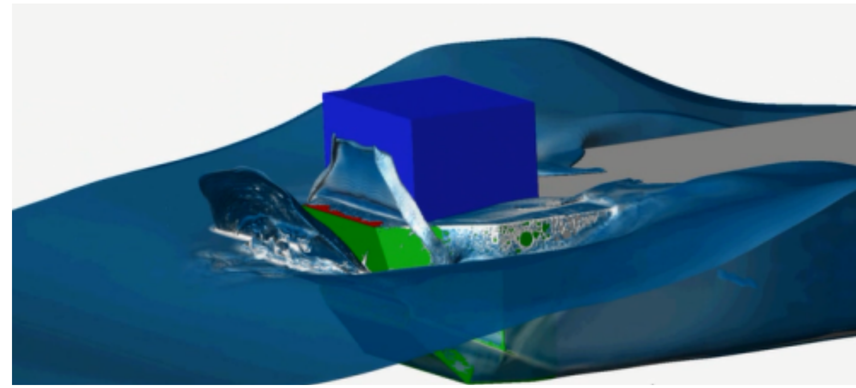
Liquid Natural Gas (LNG) sloshing

13. Industrial applications?

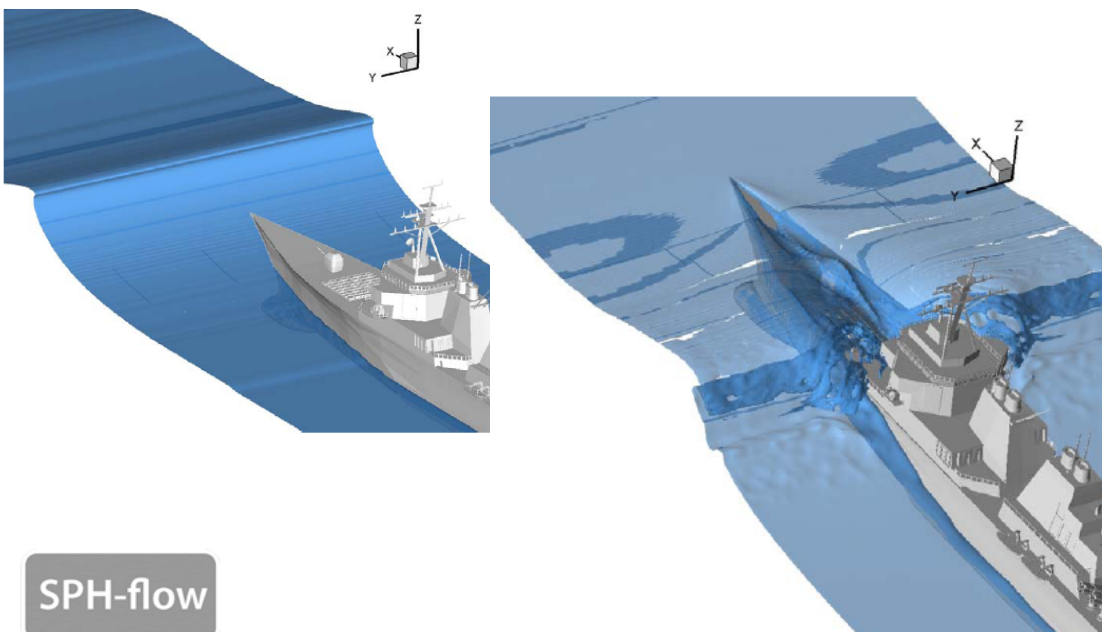
- FPSO in severe sea state

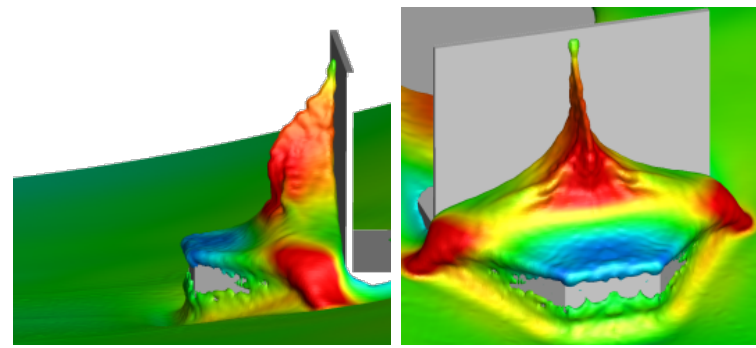
  coupling strategy:

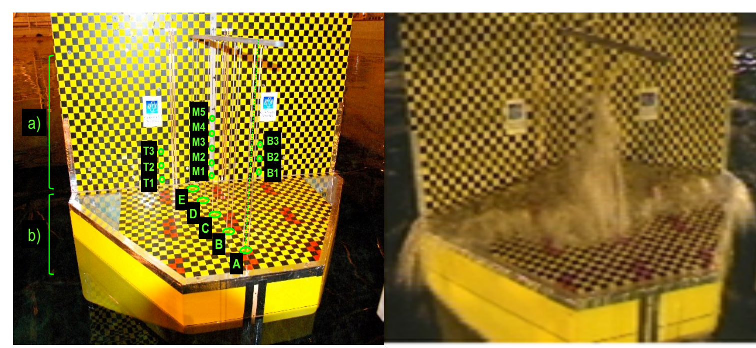




- Fregate facing a dimensioning wave

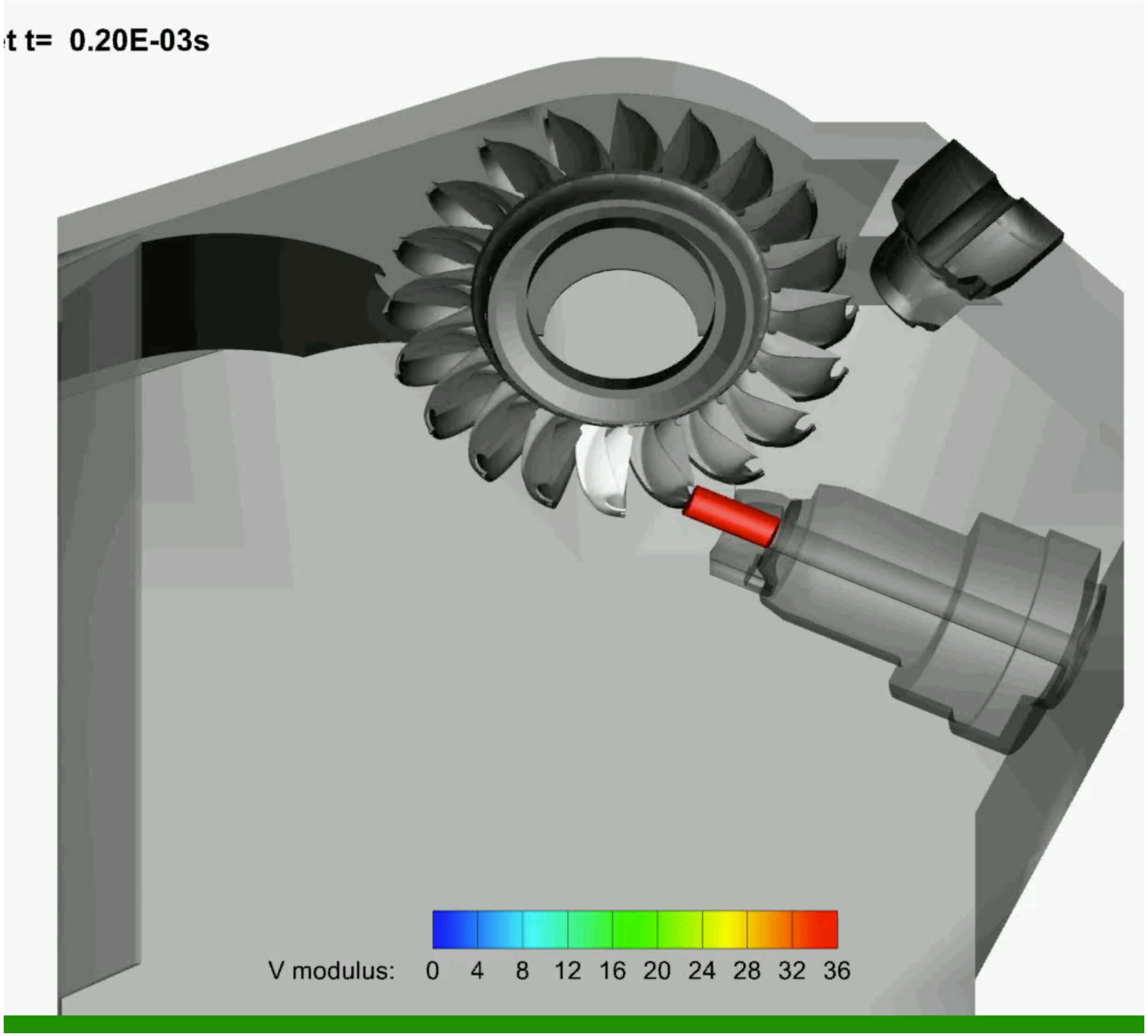






SPH-flow

13. Industrial applications?



Pelton turbine

SPH-flow

Conclusions

- A method growing for flows with complex/multiple interfaces/bodies
- The method extends also towards more and more multiphysics fields: ease to add PDEs in the system to solve and to have separated materials
- Numerical experience and understanding of the method fundamentals and numerical mechanisms is growing but a difficulty remains in terms of numerical analysis/applied maths on the method, slowing down progress towards higher-order, etc.
- A still costly method

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- Julien Candelier

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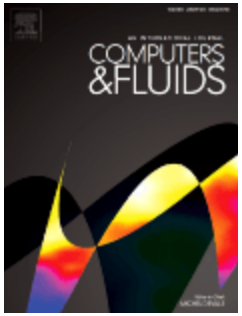
- Salvatore Marrone
- Matteo Antuono
- Andrea di Mascio



Special issue of C&F on "Theoretical, numerical and computational advances of the SPH method for solving fluid problems »

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standard pier review of C&F, managed by the 4 guest editors

C&F incentive

C&F promotes benchmarking papers (cf. C&F website), so don't hesitate to go in this direction